Geometry of Interaction for an effectful concurrent λ-calculus

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Plan

GoI as an abstract machine

GoI has several advantages as an implementation mechanism :

- Fine-grained : (ME)LL proofs nets as "functional assembly language"
- Expressive : various computational paradigm
- Comes with a semantic
- Local computations : compositional & parallelizable
- Compact runtime and object code

Challenges

Evaluation strategy	Call-by-name
Effects & Recursion	Simply typed pure λ-calculus
Concurrency	Parallelism without interaction
Efficiency	Exponential executions

Geometry of Synchronization

Geometry of Synchronization [Dal Lago, Hasuo, Faggian, Valiron, Yoshimizu] adresses several points :

- Support both for call-by-value and call-by-name
- Parallel evaluation
- Recursion

Expressive power

What about effects ?

- Add ad-hoc features to automata
- Derive generic automata [Hoshino, Muroya, Hasuo]
- Monads [Tranquilli]

Goal

Design a parallel CBV GoI machine for a concurrent effectful λ-calculus.

The main ingredients we focus on are

- A new proof nets language (assembly) that handles naturally non-determinism
- A translation of the calculus in these proof nets
- A simulation theorem

Then a straightforward extension of the GoI/GoS provides the desired machine

Plan

Concurrent λ-calculus

We use Amadio's concurrent λ-calculus with regions

Terms

variables	$x, y, \ldots$		
regions	$r, s, \ldots$		
values	V	$::=$	$x \mid * \mid \lambda x.M$
terms	M	$::=$	$V \mid M\ M \mid \text{get}(r) \mid \text{set}(r, V) \mid M \parallel M$
stores	S	$::=$	$r \Leftarrow V \mid (S \parallel S)$
programs	P	$::=$	$M \mid S \mid (P \parallel P)$

Structural rules

$$P' \parallel P = P \parallel P'$$
$$(P \parallel P') \parallel P'' = P \parallel (P' \parallel P'')$$

Reduction of the calculus

Evaluation contexts

$$
\begin{array}{lcl}
E & ::= & [.] \mid E\ M \mid V\ E \\
C & ::= & [.] \mid (C \parallel P) \mid (P \parallel C)
\end{array}
$$

Reduction rules

$$
\begin{array}{lrcl}
(\beta_v) & C[E[(\lambda x.M)\ V] & \rightarrow & C[E[M[V/x]]] \\
(\mathtt{get}) & C[E[\mathrm{get}(r)]] \parallel r \Leftarrow V & \rightarrow & C[E[V]] \parallel r \Leftarrow V \\
(\mathtt{set}) & C[E[\mathrm{set}(r, V)]] & \rightarrow & C[E[*]] \parallel r \Leftarrow V
\end{array}
$$

Reduction of the calculus

$$(\lambda x.\mathrm{get}(r))\ * \parallel \mathrm{set}(r,1) \parallel \mathrm{set}(r,2)$$

$$\downarrow$$

$$(\lambda x.\mathrm{get}(r))\ * \parallel * \parallel * \parallel r \Leftarrow 1 \parallel r \Leftarrow 2$$

$$\swarrow \qquad \searrow$$

$$1 \parallel * \parallel * \parallel r \Leftarrow 1 \parallel r \Leftarrow 2 \qquad\qquad 2 \parallel * \parallel * \parallel r \Leftarrow 1 \parallel r \Leftarrow 2$$

Reduction of the calculus

$$\mathrm{set}(r,1);\mathrm{set}(r,2);\mathrm{get}(r) = \lambda x.(\lambda y.(\mathrm{get}(r))\ \mathrm{set}(r,2))\ \mathrm{set}(r,1)$$

$$\downarrow$$

$$\mathrm{set}(r,2);\mathrm{get}(r) \parallel r \Leftarrow 1$$

$$\downarrow$$

$$\mathrm{get}(r) \parallel r \Leftarrow 1 \parallel r \Leftarrow 2$$

$$\swarrow \qquad \searrow$$

$$1 \parallel r \Leftarrow 1 \parallel r \Leftarrow 2 \qquad\qquad 2 \parallel r \Leftarrow 1 \parallel r \Leftarrow 2$$

Type system

Types

effects	e, e'	$=$	$\{r_1, \ldots, r_n\}$
types	α	$::=$	$\mathbf{B} \mid A$
values types	A	$::=$	$\mathsf{Unit} \mid A \xrightarrow{e} \alpha \mid \mathsf{Reg}_r A$
variable contexts	Γ	$::=$	$x_1 : A_1, \ldots, x_n : A_n$
region contexts	R	$::=$	$r_1 : A_1, \ldots, r_n : A_n$

Judgement

$$R; \Gamma \vdash P : (\alpha, e)$$

Termination

Landin's trick gives a fixpoint

$$\begin{array}{rc} & \mathsf{set}(r, \lambda x.\mathsf{get}(r)\ x); \mathsf{get}(r) * \\ \to & \mathsf{get}(r) * \parallel r \Leftarrow \lambda x.\mathsf{get}(r)\ x \\ \to & (\lambda x.\mathsf{get}(r)\ x) * \parallel r \Leftarrow \lambda x.\mathsf{get}(r)\ x \\ \to & \mathsf{get}(r) * \parallel r \Leftarrow \lambda x.\mathsf{get}(r)\ x \\ \to & \ldots \end{array}$$

A natural constraint to recover termination is stratification

Stratification conditions

$$\frac{}{\emptyset \vdash} \qquad \frac{R \vdash A \qquad r \notin \mathrm{dom}(R)}{R, r : A \vdash}$$

$$\frac{R \vdash}{R \vdash \mathit{Unit}} \qquad \frac{R \vdash}{R \vdash \mathbf{B}}$$

$$\frac{R \vdash A \qquad R \vdash \alpha \qquad e \subseteq \mathrm{dom}(R)}{R \vdash A \xrightarrow{e} \alpha} \qquad \frac{R \vdash \qquad r : A \in R}{R \vdash \mathrm{Reg}_r A}$$

Stratification as conditions

For $R = r_1 : A_1, \ldots, r_n : A_n$, the following are equivalent

- R is stratified (well formed)
- $\mathrm{Eff}(\alpha) \subseteq \mathrm{dom}(R)$ and
  $(\forall i \ : 1 \leq i \leq n),\ \mathrm{Eff}(A_i) \subseteq \{r_{i-1}, \ldots, r_1\}$
- The following equation system is solvable
  - $\mathrm{Unit}^\bullet = !1$
  - $\mathbf{B}^\bullet = !1$
  - $(A \overset{\{s_1,\ldots,s_m\}}{\longrightarrow} \alpha)^\bullet =$
    $!((A^\bullet \otimes X_{s_1} \ldots \otimes X_{s_m}) \multimap (X_{s_1} \otimes \ldots \otimes X_{s_m} \otimes \alpha^\bullet))$
  - $X_{r_i} = {A_i}^\bullet$

Properties

Subject reduction

Termination

Progress

If $\Gamma \vdash M : (\alpha, e)$, then $M \rightarrow^* N_1 \parallel \ldots \parallel N_n$ where N_i is either a value or of the form $E[\text{get}(r)]$

Plan

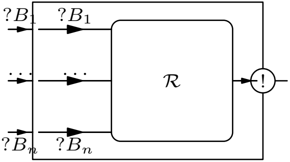
Differential linear logic

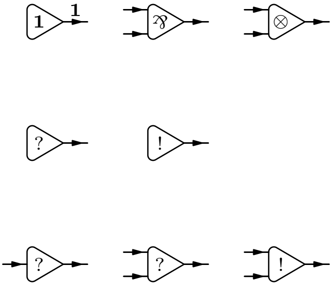
Differential linear logic [Ehrhard, Regnier] is an extension of linear logic with symmetric dual operators :

- codereliction : $A \to !A$
- coweakening : $1 \to !A$
- cocontraction : $!A \otimes !A \to !A$

We only need cocontraction and coweakening.
Reductions rules are naturally non-deterministic : we must consider **formal sums** of proofs.

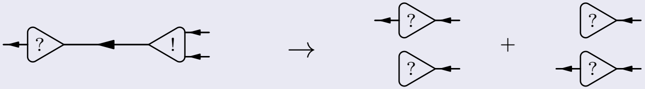
Differential interaction nets





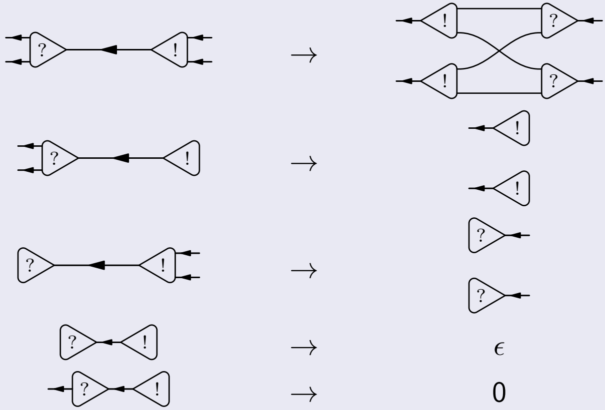
Reduction rules

Non-deterministic reduction



Reduction rules

Co-reductions



Plan

Using monads

Encoding for type and effect system :

$$\Gamma \vdash M : (\alpha, e) \text{ with } e = \{r_1, \ldots, r_n\}$$

$S = R_1 \times \ldots \times R_n$, then

$$\begin{aligned} T_e(\alpha) &= S \to S \times \alpha \\ T_e(A \xrightarrow{e} \alpha) &= A \to (S \to (S \times \alpha)) \cong A \times S \to S \times \alpha \end{aligned}$$

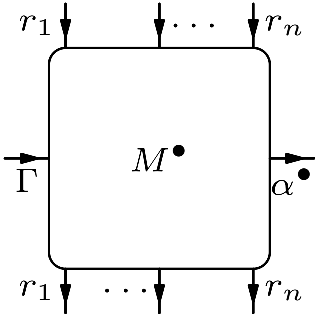
Stratification as conditions

For $R = r_1 : A_1, \ldots, r_n : A_n$, the following are equivalent

- R is stratified (well formed)
- $\mathrm{Eff}(\alpha) \subseteq \mathrm{dom}(R)$ and $(\forall i \ : 1 \leq i \leq n),\ \mathrm{Eff}(A_i) \subseteq \{r_{i-1}, \ldots, r_1\}$
- The following equation system is solvable
  - $\mathrm{Unit}^\bullet = !1$
  - $\mathbf{B}^\bullet = !1$
  - $(A \overset{\{s_1,\ldots,s_m\}}{\rightarrow} \alpha)^\bullet = !((A^\bullet \otimes X_{s_1} \ldots \otimes X_{s_m}) \multimap (X_{s_1} \otimes \ldots \otimes X_{s_m} \otimes \alpha^\bullet))$
  - $X_{r_i} = A_i^\bullet$

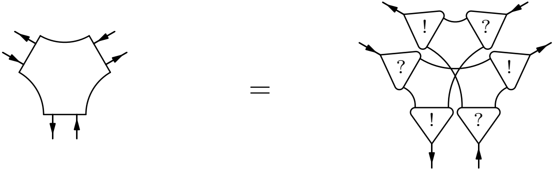
Idea

We translate a term $\Gamma \vdash M : (\alpha, e)$ to a net $M^\bullet$



Building blocks

[Ehrhard, Laurent]



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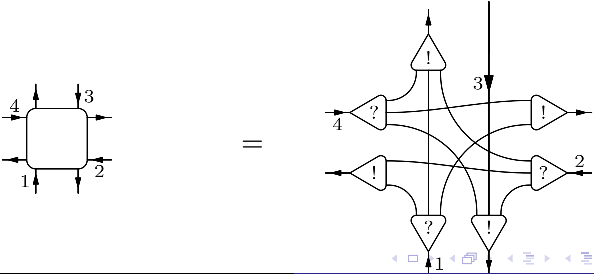

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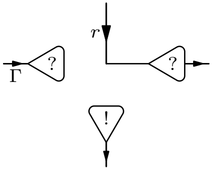
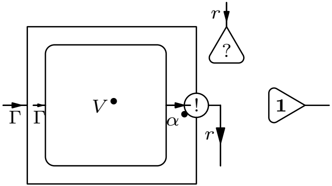
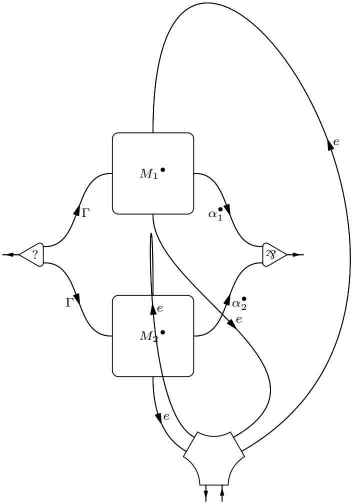
Translation





Translation



Cycles ?

Two kind of cycles :

First type

$$\lambda x.(\text{set}(r, x))\ \text{get}(r)$$

Fix : use custom communication area

Second type

$$\lambda x.(\text{set}(r, x))\ \text{get}(r) \parallel \lambda x.(\text{set}(r, x))\ \text{get}(r)$$

Seems to be "invisible" to GoI

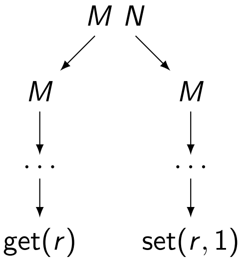
Simulation

Theorem

Let $\Gamma \vdash M : (\alpha, e)$ be a term

- If $M \to N$ by (β_v) or $(\mathtt{set})$ then $M^\bullet \to N^\bullet$
- If $M \to N_i$ by $(\mathtt{get})$ then $M^\bullet \to M_0^\bullet + \sum_i N_i^\bullet$

From shared memory to messages



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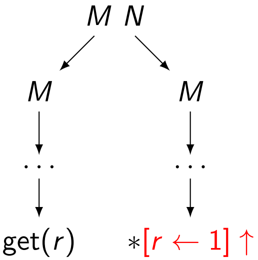

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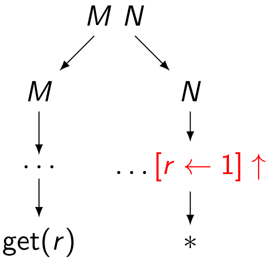

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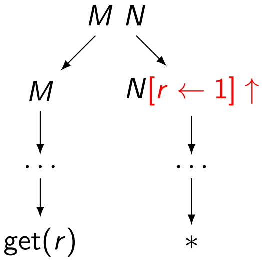

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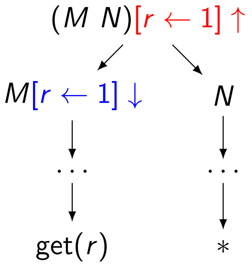

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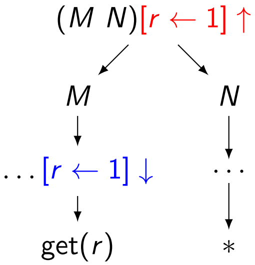

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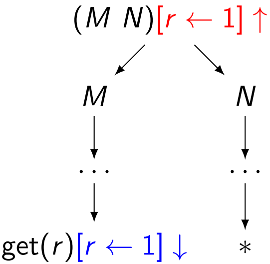

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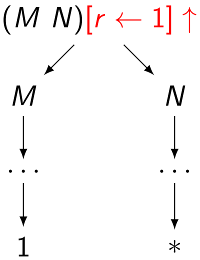

Plan

What we've done

- Translation and simulation of a concurrent effectful λ-calculus in (a subset of) differential interaction nets
- Turned a global shared-memory model to a local message-passing model
- A [soon-to-be] parallel CBV token machine

Thank you !