

The relational semantics is injective for Multiplicative Exponential Linear Logic

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Five natural problems about MELL

- **Canonicity of proofs**
- Completeness (aka injectivity)
- Principal typing
- Taylor expansion injectivity
- Confluence

The problem of canonicity of proofs

Intuitionistic sequent calculus (LJ)	Natural deduction
MELL sequent calculus	?

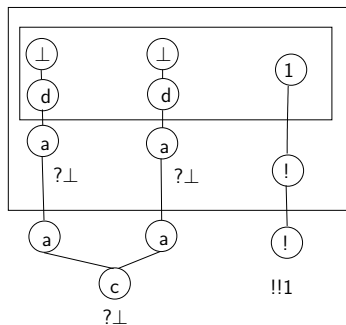
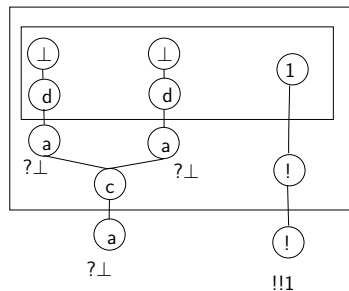
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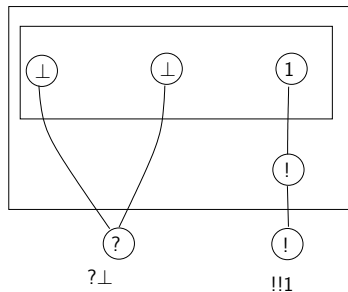
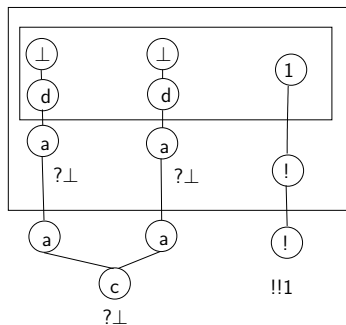
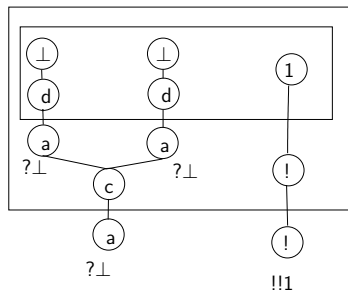
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Non-canonicity of Girard proof-nets



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The problem of completeness

Theorem. (Friedman 1975) There exists an interpretation $\llbracket \cdot \rrbracket_-$ of the simply typed λ -calculus in the category **Set** of sets and functions such that, for any terms $v, u : \tau$, for any suitable environment ρ , we have

$$v \simeq_{\beta\eta} u \Leftrightarrow \llbracket v \rrbracket_{\rho} = \llbracket u \rrbracket_{\rho}$$

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Notice that there are different cut-free proof-nets that have the same *coherence* interpretation: the coherence semantics is *not* complete for MELL. (Tortora de Falco)

The relational semantics

Grammar of types:

$$\langle T \rangle ::= \mathbf{1} \mid \perp \mid (\langle T \rangle \otimes \langle T \rangle) \mid (\langle T \rangle \wp \langle T \rangle) \mid !\langle T \rangle \mid ?\langle T \rangle$$

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Interpretation of types:

$$\llbracket \mathbf{1} \rrbracket = \{*\} = \llbracket \perp \rrbracket$$

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The relational semantics

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Interpretation of proof-nets: if π has conclusions of types C_1, \dots, C_m , then $\llbracket \pi \rrbracket \subseteq \prod_{j=1}^m \llbracket C_j \rrbracket$ is the set of the *results* of the *experiments* of π .

MLL experiments

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An experiment of π is a labelling e of the ports of π such that if $p : A$ is at depth 0, then $e(p) \in \llbracket A \rrbracket$:

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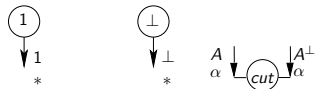
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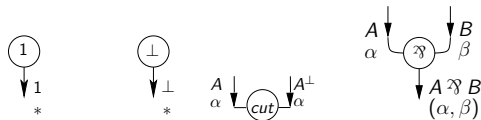
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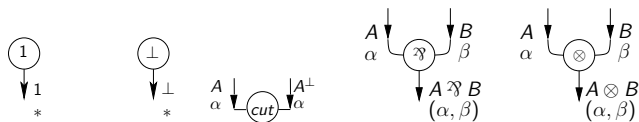
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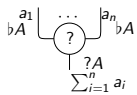
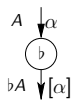
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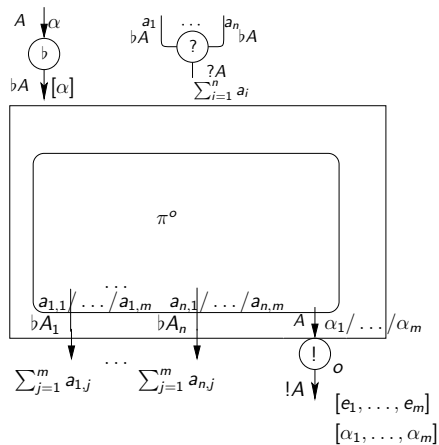


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The problem of principal typing

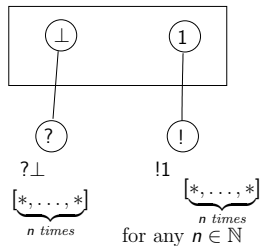
In intersection types systems of λ -calculus, from one (well-chosen) typing, one can recover all its intersection typings.
(Coppo-Dezani-Venneri)

The problem of principal typing

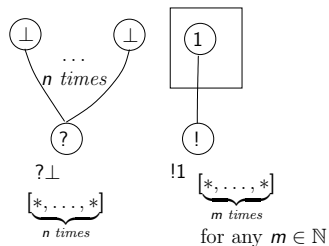
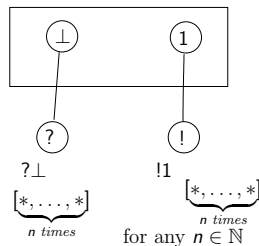
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Can we have a similar result for MELL, i.e:
for any MELL proof-net, does there exist an experiment whose result allows to recover all the results of its experiments?

No principal typing for MELL



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A weak principal typing property

I will show: for any MELL proof-net, there exist *two* experiments whose results allow to recover all the results of its experiments.

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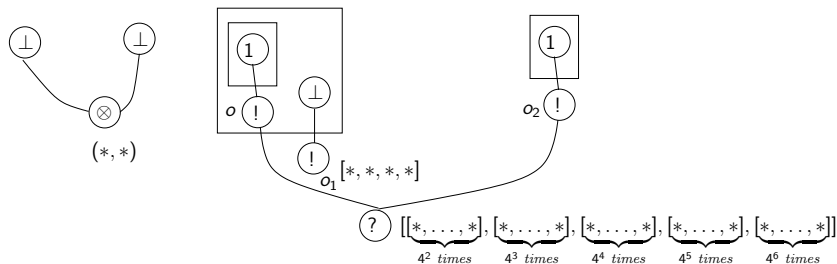
- One experiment is a 1-experiment: it allows to bound the maximal arity of the contractions and the number of boxes by some $k \in \mathbb{N}$.
- A second experiment is a *k-heterogeneous experiment*.

k -heterogenous experiments

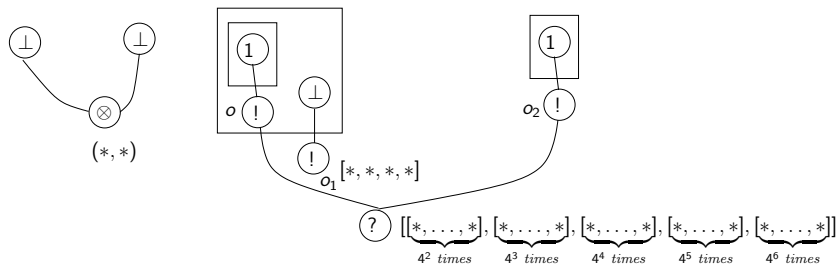
A k -heterogenous experiment is an experiment such that:

- all the positive multisets in the result have cardinality k^j for some $j > 0$
- and two different occurrences of positive multisets in the result have different cardinalities.

Heterogeneous experiments are not determined by their results



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Five different 4-heterogeneous experiments give the same result:

- a 4-heterogeneous experiment takes 16 copies of the box o_2
- a 4-heterogeneous experiment takes 64 copies of the box o_2

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The problem of Taylor expansion injectivity

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Do two different MELL proof-nets have two different Taylor expansions?

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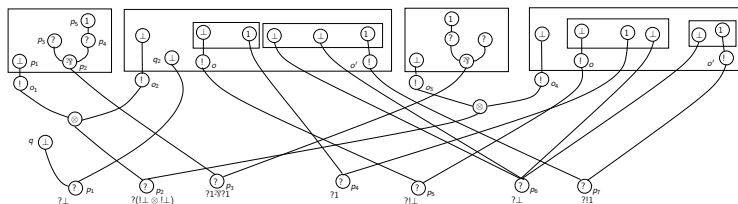
The Taylor expansion of a MELL proof-net is an infinite series of its linear approximations.

Do two different MELL proof-nets have two different Taylor expansions?

A basic remark: for a *cut-free* MELL proof-net, results of experiments are essentially the differential nets of its Taylor expansion.

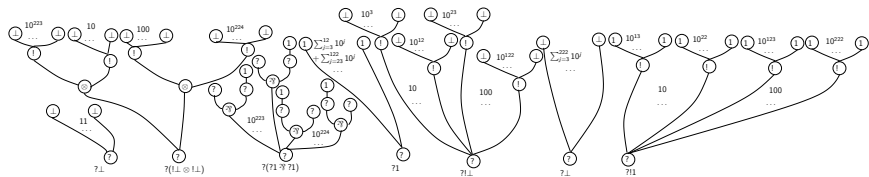
So, since MELL proof-nets are normalizable, the problem of completeness is equivalent to the problem of Taylor expansion injectivity for cut-free MELL proof-nets.

An example



There exists a 10-heterogeneous experiment f of this proof-net π s.t.

- $f^\#(o_1) = \{10^{223}\}$
- $f^\#(o_2) = \{10\}$
- $f^\#(o_3) = \{10^{224}\}$
- $f^\#(o_4) = \{100\}$
- $f^\#((o_2, o)) = \{10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}, 10^{11}, 10^{12}\}$
- $f^\#((o_2, o')) = \{10^{13}, 10^{14}, 10^{15}, 10^{16}, 10^{17}, 10^{18}, 10^{19}, 10^{20}, 10^{21}, 10^{22}\}$
- $f^\#((o_4, o)) = \{10^{23}, \dots, 10^{122}\}$
- $f^\#((o_4, o')) = \{10^{123}, \dots, 10^{222}\}$

$\mathcal{T}(f)[0]$ 

The algorithm rebuilding the proof-net

More generally, $\mathcal{T}(f)[i]$ is the differential net (with boxes) that consists in expanding all the boxes of depth $\geq i$ as many times as f duplicated them.

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The $i + 1$ -th step consists in rebuilding $\mathcal{T}(f)[i + 1]$ from $\mathcal{T}(f)[i]$.

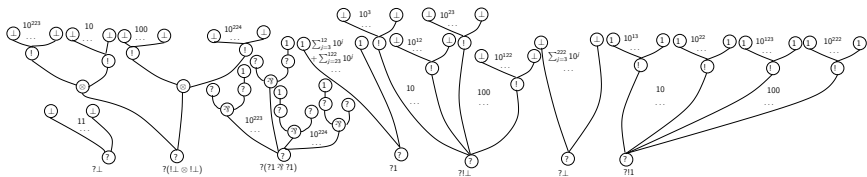
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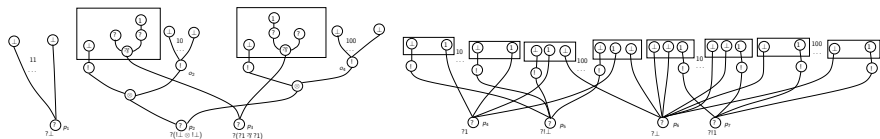
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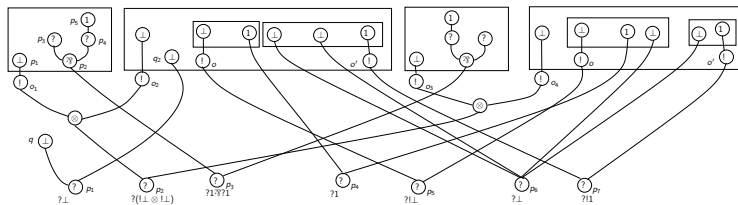
The first step of the algorithm consists in rebuilding $\mathcal{T}(f)[1]$ from $\mathcal{T}(f)[0]$.

The $i + 1$ -th step consists in rebuilding $\mathcal{T}(f)[i + 1]$ from $\mathcal{T}(f)[i]$.

Now, since $\mathcal{T}(f)[depth(\pi)] = \pi$, the problem of rebuilding π is reduced to the problem of rebuilding $\mathcal{T}(f)[i + 1]$ from $\mathcal{T}(f)[i]$.

$\mathcal{T}(f)[0]$ 

$\mathcal{T}(f)[1]$ 

$\mathcal{T}(f)[2]$ 

From $\mathcal{T}(e)[i]$ to $\mathcal{T}(e)[i + 1]$ (1)

Definition. Let π be a MELL proof-net. Let $k > 1$. Let e be a k -heterogeneous experiment of π . For any $i, j \in \mathbb{N}$, we define, by induction on i , $\mathcal{M}_i(e) \subseteq \mathbb{N} \setminus \{0\}$ and $(m_{i,j}(e))_{j \in \mathbb{N}} \in \{0, \dots, k-1\}^{\mathbb{N}}$ as follows:

We set $\mathcal{M}_0(e) = \bigcup_{o \in \mathcal{B}(\pi)} \{j \in \mathbb{N}; k^j \in e^\#(o)\}$.

We write $\text{Card}(\mathcal{M}_i(e))$ in base k :

$$\text{Card}(\mathcal{M}_i(e)) = \sum_{j \in \mathbb{N}} m_{i,j}(e) \cdot k^j$$

and we set $\mathcal{M}_{i+1}(e) = \{j > 0; m_{i,j}(e) \neq 0\}$.

For any $i \in \mathbb{N}$, we set $\mathcal{N}_i(e) = \mathcal{M}_i(e) \setminus \mathcal{M}_{i+1}(e)$.

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Example

We have $\mathcal{M}_0(f) = \{1, \dots, 224\}$. So we have

$\text{Card}(\mathcal{M}_0(f)) = 4 + 2 \cdot 10^1 + 2 \cdot 10^2$, hence $\mathcal{M}_1(f) = \{1, 2\}$ and

$\mathcal{N}_0(f) = \{3, \dots, 224\}$. We have $\text{Card}(\mathcal{M}_1(f)) = 2 < 10$, hence

$\mathcal{M}_2(f) = \emptyset$ and $\mathcal{N}_1(f) = \{1, 2\}$.

From $\mathcal{T}(e)[i]$ to $\mathcal{T}(e)[i + 1]$ (2)

Definition. Let π be a differential net, $k > 1$, $j > 0$. The set $\mathcal{K}_{k,j}(\pi)$ is the set of ports p of π at depth 0 such that the j -th digit of the arity of p in base k is non-null.

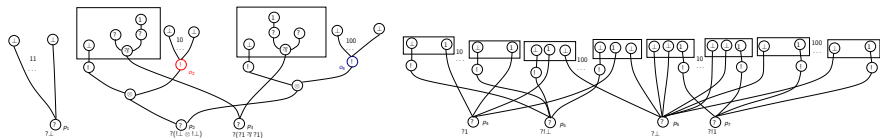
We denote by $!_{e,i}$ the bijection

$$\bigcup_{o \in \mathcal{B}^{\geq i}(\pi)} \{\log_k(m); m \in e^\#(o)\} \rightarrow \mathcal{P}_0^!(\mathcal{T}(e)[i]) \setminus \mathcal{B}_0(\mathcal{T}(e)[i])$$

s.t., for any $j \in \bigcup_{o \in \mathcal{B}^{\geq i}(\pi)} \{\log_k(m); m \in e^\#(o)\}$, we have $(a_{\mathcal{T}(e)[i]} \circ !_{e,i})(j) = k^j$.

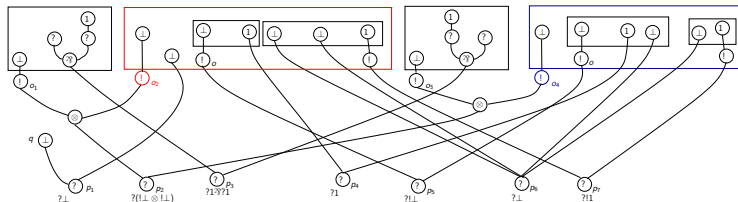
Proposition. Let π be a MELL proof-net. Let $k > \text{Card}(\mathcal{B}(\pi))$, $\text{cosize}(\pi)$. Let e be a k -heterogeneous experiment of π and let $i \in \mathbb{N}$. Then we have $\mathcal{B}_0^{\leq i}(\mathcal{T}(e)[i + 1]) = !_{e,i}[\mathcal{N}_{i,e}]$. Furthermore, for any $j \in \mathcal{N}_i(e)$, we have $\text{im}(b_{\mathcal{T}(e)[i+1]}(!_{e,i}(j))) = \mathcal{K}_{k,j}(\mathcal{T}(e)[i])$.

Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: $!_{f,1}[\mathcal{N}_1(f)]$



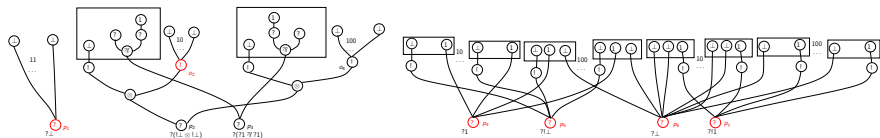
We have $!_{f,1}[\mathcal{N}_1(f)] = \{o_2, o_4\}$.

Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$:
 $\mathcal{B}_0^{-1}(\mathcal{T}(f)[2])$



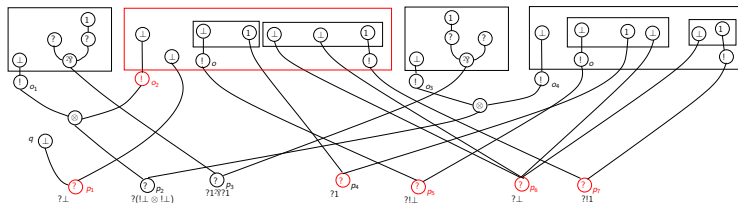
We have $\mathcal{B}_0^{-1}(\mathcal{T}(f)[2]) = \{\alpha_2, \alpha_4\}$.

Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: $\mathcal{K}_{10,1}(\mathcal{T}(f)[1])$



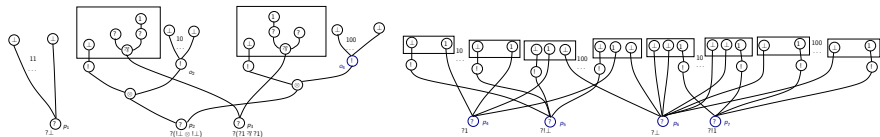
We have $\mathcal{K}_{10,1}(\mathcal{T}(f)[1]) = \{p_1, p_4, p_5, p_6, p_7, o_2\}$

Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$:
 $\text{im}(b_{\mathcal{T}(f)[2]})(o_2)$



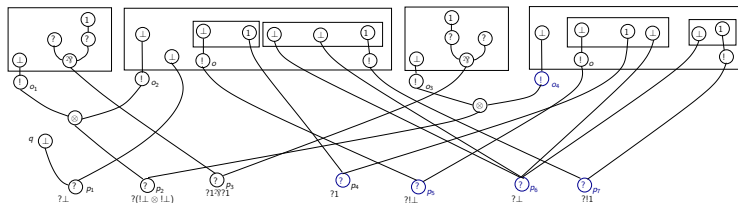
We have $\text{im}(b_{\mathcal{T}(f)[2]})(o_2) = \{p_1, p_4, p_5, p_6, p_7, o_2\}$.

Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$:
 $\mathcal{K}_{10,2}(\mathcal{T}(f)[1])$



We have $\mathcal{K}_{10,2}(\mathcal{T}(f)[1]) = \{p_4, p_5, p_6, p_7, o_4\}$

Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$:
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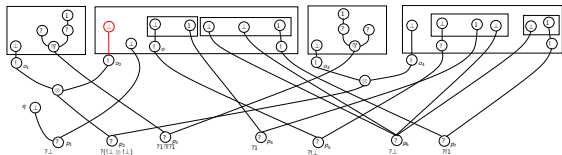
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From $\mathcal{T}(e)[i]$ to $\mathcal{T}(e)[i + 1]$ (3)

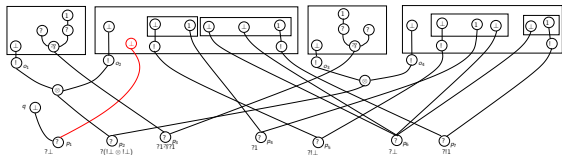
Proposition. Let π be a MELL proof-net. Let $k > \text{Card}(\mathcal{B}(\pi))$, $\text{cosize}(\pi)$. Let e be a k -injective pseudo-experiment of π . Let $i \in \mathbb{N}$. Let $j_0 \in \mathcal{N}_i(e)$. We set $\mathcal{T}_{j_0} = \mathcal{S}_{\mathcal{T}(e)[i]}^k(\mathcal{K}_{k,j_0}(\mathcal{T}(e)[i]))$. Let $U \in \mathcal{T}_{j_0}$. Let $(m_j)_{j \in \mathbb{N}} \in \{0, \dots, k-1\}^{\mathbb{N}}$ such that $\text{Card}(\{T' \in \mathcal{T}_{j_0}; T' \equiv U\}) = \sum_{j \in \mathbb{N}} m_j \cdot k^j$. Then we have

$$m_{j_0} = \text{Card} \left(\{T' \in \mathcal{C}^k(\mathcal{B}_{\mathcal{T}(e)[i+1]}(!_{e,i}(j_0))); \mathcal{I}_{e,i,o}(T', \overline{U}) \neq \emptyset\} \right)$$

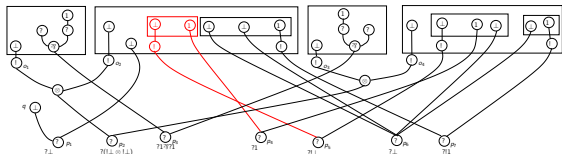
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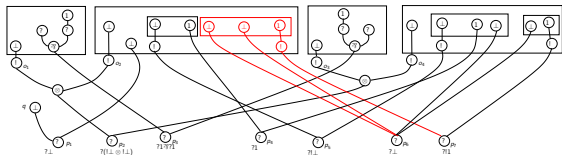
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$:
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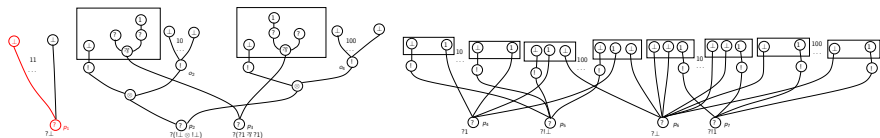
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$:
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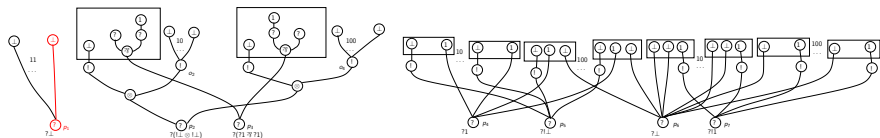
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$:
 $\mathcal{C}^{10}(B_{\mathcal{T}(f)[2]}(!_{e,i}(1)))$



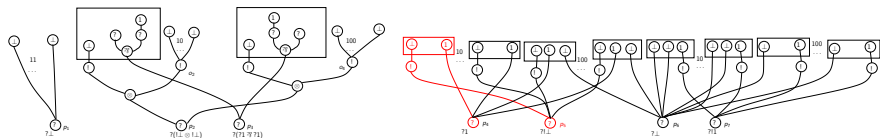
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



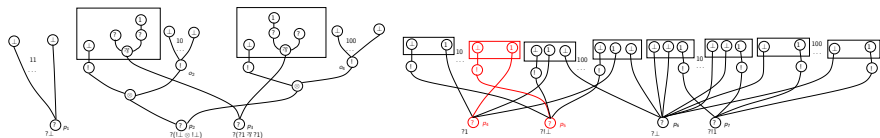
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



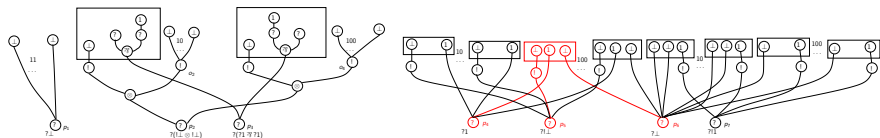
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



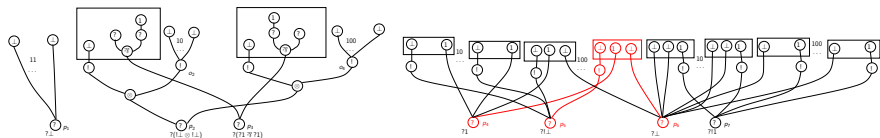
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



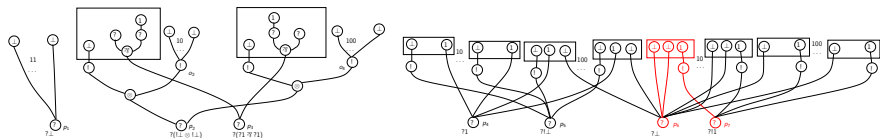
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



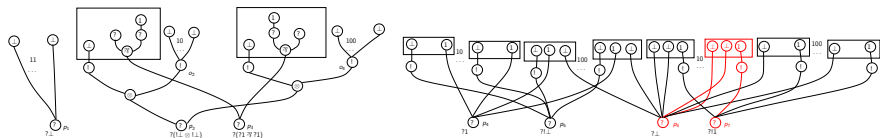
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



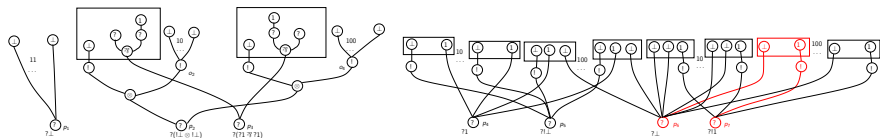
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



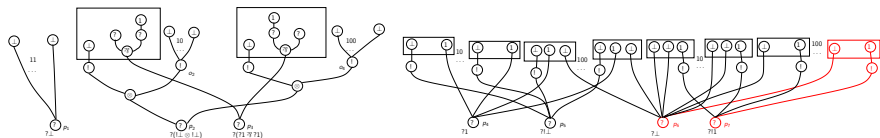
Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$: \mathcal{T}_1



Confluence

Let π_1 and π_2 cut-free s.t. $\pi \rightarrow^* \pi_1$ and $\pi \rightarrow^* \pi_2$. We have $[[\pi_1]] = [[\pi_2]]$, hence $\pi_1 = \pi_2$.

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 - finiteness spaces (Ehrhard)
 - weighted sets (Amini-Ehrhard)
- A weak principal typing property holds for normalizable proof-nets.
- The Taylor expansion is injective for cut-free proof-nets; it seems that the same proof should work in presence of cuts.