

# The relational semantics is injective for Multiplicative Exponential Linear Logic

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# Five natural problems about MELL

- **Canonicity of proofs**
- Completeness (aka injectivity)
- Principal typing
- Taylor expansion injectivity
- Confluence

# The problem of canonicity of proofs

Intuitionistic sequent calculus (LJ)	Natural deduction
MELL sequent calculus	?

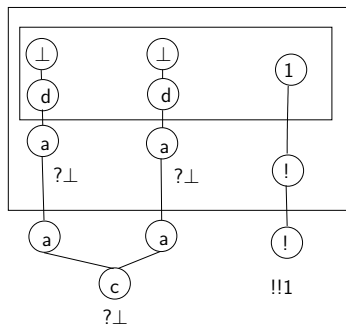
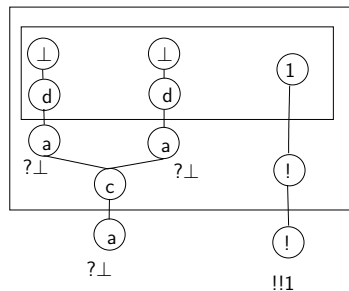
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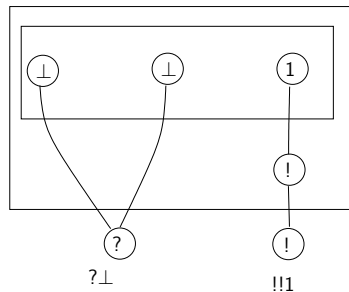
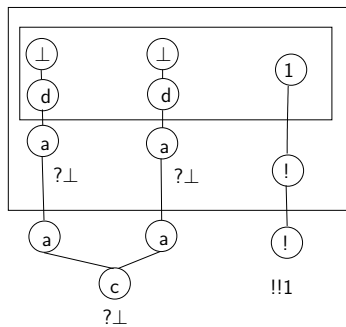
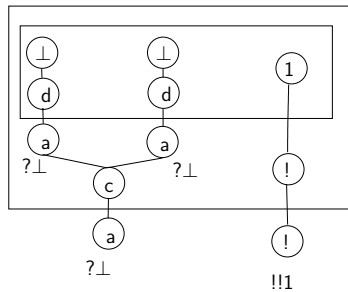
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# Non-canonicity of Girard proof-nets



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## The problem of completeness

**Theorem.** (Friedman 1975) There exists an interpretation  $\llbracket \cdot \rrbracket_-$  of the simply typed  $\lambda$ -calculus in the category **Set** of sets and functions such that, for any terms  $v, u : \tau$ , for any suitable environment  $\rho$ , we have

$$v \simeq_{\beta\eta} u \Leftrightarrow \llbracket v \rrbracket_{\rho} = \llbracket u \rrbracket_{\rho}$$

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Notice that there are different cut-free proof-nets that have the same *coherence* interpretation: the coherence semantics is *not* complete for MELL. (Tortora de Falco)

# The relational semantics

Grammar of types:

$$\langle T \rangle ::= \mathbf{1} \mid \perp \mid (\langle T \rangle \otimes \langle T \rangle) \mid (\langle T \rangle \wp \langle T \rangle) \mid !\langle T \rangle \mid ?\langle T \rangle$$

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Interpretation of types:

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# The relational semantics

Grammar of types:

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Interpretation of types:

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Interpretation of proof-nets: if  $\pi$  has conclusions of types  $C_1, \dots, C_m$ , then  $\llbracket \pi \rrbracket \subseteq \prod_{j=1}^m \llbracket C_j \rrbracket$  is the set of the *results* of the *experiments* of  $\pi$ .

# MLL experiments



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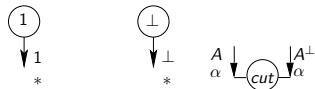
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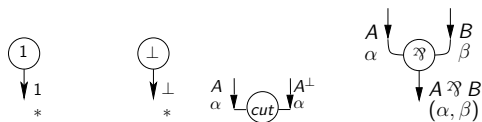
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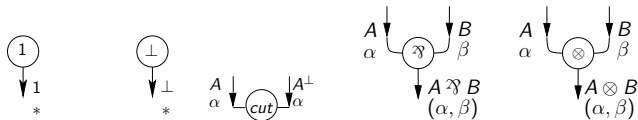
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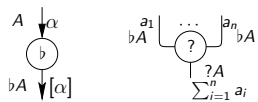
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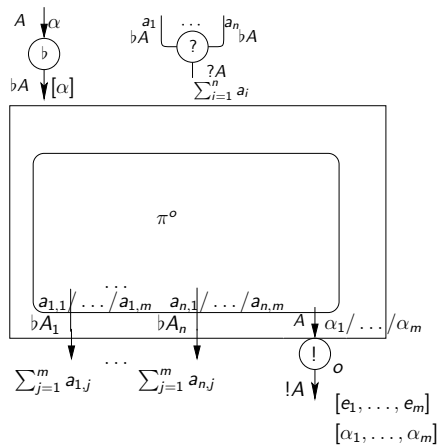
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# The problem of principal typing

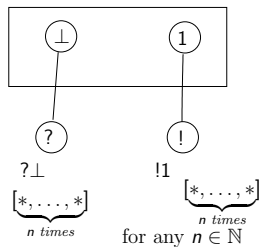
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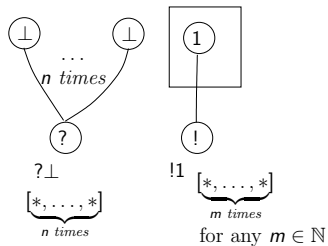
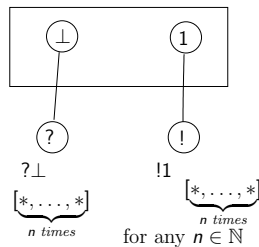
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Can we have a similar result for MELL, i.e:  
for any MELL proof-net, does there exist an experiment whose result allows to recover all the results of its experiments?

# No principal typing for MELL



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## A weak principal typing property

I will show: for any MELL proof-net, there exist *two* experiments whose results allow to recover all the results of its experiments.



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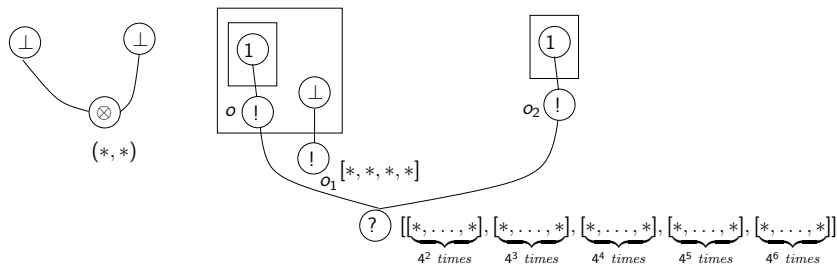
- One experiment is a 1-experiment: it allows to bound the maximal arity of the contractions and the number of boxes by some  $k \in \mathbb{N}$ .
- A second experiment is a *k-heterogeneous experiment*.

## $k$ -heterogenous experiments

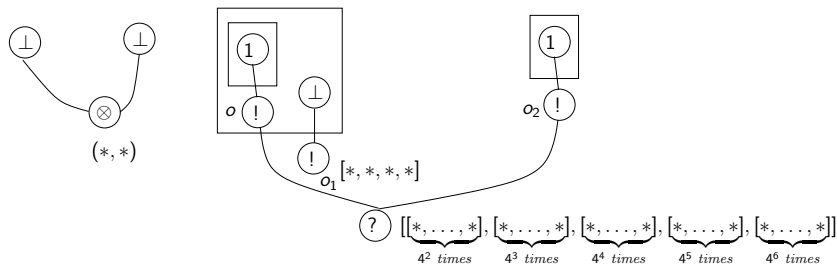
A  $k$ -heterogenous experiment is an experiment such that:

- all the positive multisets in the result have cardinality  $k^j$  for some  $j > 0$
- and two different occurrences of positive multisets in the result have different cardinalities.

# Heterogeneous experiments are not determined by their results



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Five different 4-heterogeneous experiments give the same result:

- a 4-heterogeneous experiment takes 16 copies of the box  $o_2$
- a 4-heterogeneous experiment takes 64 copies of the box  $o_2$

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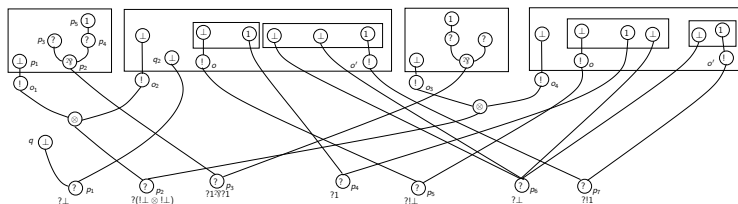
The Taylor expansion of a MELL proof-net is an infinite series of its linear approximations.

Do two different MELL proof-nets have two different Taylor expansions?

A basic remark: for a *cut-free* MELL proof-net, results of experiments are essentially the differential nets of its Taylor expansion.

So, since MELL proof-nets are normalizable, the problem of completeness is equivalent to the problem of Taylor expansion injectivity for cut-free MELL proof-nets.

## An example



There exists a 10-heterogeneous experiment  $f$  of this proof-net  $\pi$  s.t.

- $f^\#(o_1) = \{10^{223}\}$
- $f^\#(o_2) = \{10\}$
- $f^\#(o_3) = \{10^{224}\}$
- $f^\#(o_4) = \{100\}$
- $f^\#((o_2, o)) = \{10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}, 10^{11}, 10^{12}\}$
- $f^\#((o_2, o')) = \{10^{13}, 10^{14}, 10^{15}, 10^{16}, 10^{17}, 10^{18}, 10^{19}, 10^{20}, 10^{21}, 10^{22}\}$
- $f^\#((o_4, o)) = \{10^{23}, \dots, 10^{122}\}$
- $f^\#((o_4, o')) = \{10^{123}, \dots, 10^{222}\}$



## The algorithm rebuilding the proof-net

More generally,  $\mathcal{T}(f)[i]$  is the differential net (with boxes) that consists in expanding all the boxes of depth  $\geq i$  as many times as  $f$  duplicated them.

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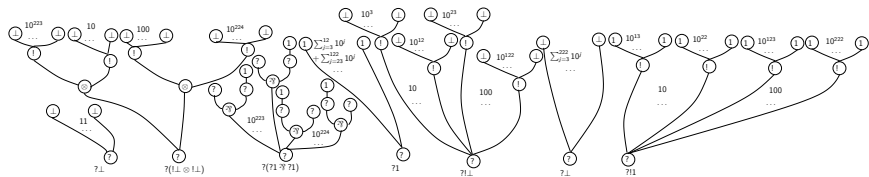
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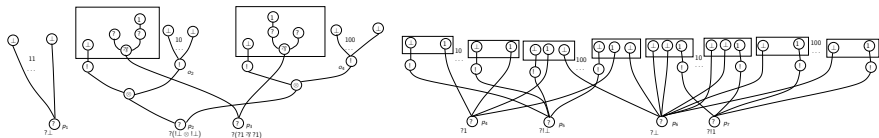
Now, since  $\mathcal{T}(f)[\text{depth}(\pi)] = \pi$ , the problem of rebuilding  $\pi$  is reduced to the problem of rebuilding  $\mathcal{T}(f)[i + 1]$  from  $\mathcal{T}(f)[i]$ .

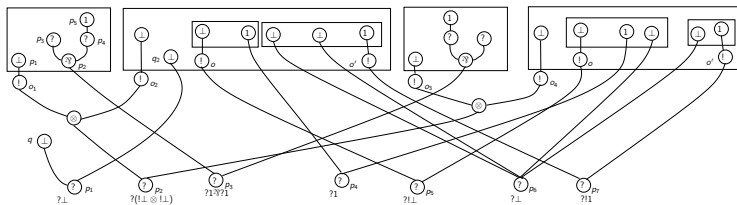
$\mathcal{T}(f)[0]$





# $\mathcal{T}(f)[1]$



$\mathcal{T}(f)[2]$ 

## From $\mathcal{T}(e)[i]$ to $\mathcal{T}(e)[i + 1]$ (1)

**Definition.** Let  $\pi$  be a MELL proof-net. Let  $k > 1$ . Let  $e$  be a  $k$ -heterogeneous experiment of  $\pi$ . For any  $i, j \in \mathbb{N}$ , we define, by induction on  $i$ ,  $\mathcal{M}_i(e) \subseteq \mathbb{N} \setminus \{0\}$  and  $(m_{i,j}(e))_{j \in \mathbb{N}} \in \{0, \dots, k-1\}^{\mathbb{N}}$  as follows:

We set  $\mathcal{M}_0(e) = \bigcup_{o \in \mathcal{B}(\pi)} \{j \in \mathbb{N}; k^j \in e^\#(o)\}$ .

We write  $\text{Card}(\mathcal{M}_i(e))$  in base  $k$ :

$$\text{Card}(\mathcal{M}_i(e)) = \sum_{j \in \mathbb{N}} m_{i,j}(e) \cdot k^j$$

and we set  $\mathcal{M}_{i+1}(e) = \{j > 0; m_{i,j}(e) \neq 0\}$ .

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### Example

We have  $\mathcal{M}_0(f) = \{1, \dots, 224\}$ . So we have

$\text{Card}(\mathcal{M}_0(f)) = 4 + 2 \cdot 10^1 + 2 \cdot 10^2$ , hence  $\mathcal{M}_1(f) = \{1, 2\}$  and

$\mathcal{N}_0(f) = \{3, \dots, 224\}$ . We have  $\text{Card}(\mathcal{M}_1(f)) = 2 < 10$ , hence

$\mathcal{M}_2(f) = \emptyset$  and  $\mathcal{N}_1(f) = \{1, 2\}$ .

## From $\mathcal{T}(e)[i]$ to $\mathcal{T}(e)[i + 1]$ (2)

**Definition.** Let  $\pi$  be a differential net,  $k > 1$ ,  $j > 0$ . The set  $\mathcal{K}_{k,j}(\pi)$  is the set of ports  $p$  of  $\pi$  at depth 0 such that the  $j$ -th digit of the arity of  $p$  in base  $k$  is non-null.

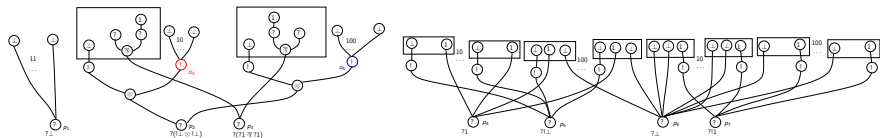
We denote by  $!_{e,i}$  the bijection

$$\bigcup_{o \in \mathcal{B}^{\geq i}(\pi)} \{\log_k(m); m \in e^\#(o)\} \rightarrow \mathcal{P}_0^!(\mathcal{T}(e)[i]) \setminus \mathcal{B}_0(\mathcal{T}(e)[i])$$

s.t., for any  $j \in \bigcup_{o \in \mathcal{B}^{\geq i}(\pi)} \{\log_k(m); m \in e^\#(o)\}$ , we have  $(a_{\mathcal{T}(e)[i]} \circ !_{e,i})(j) = k^j$ .

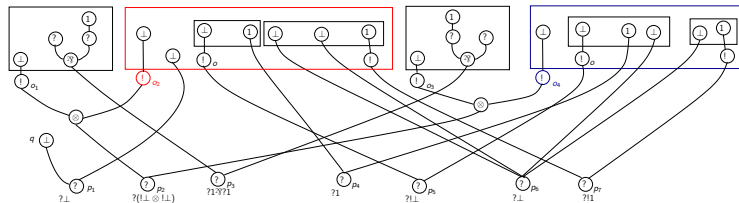
**Proposition.** Let  $\pi$  be a MELL proof-net. Let  $k > \text{Card}(\mathcal{B}(\pi))$ ,  $\text{cosize}(\pi)$ . Let  $e$  be a  $k$ -heterogeneous experiment of  $\pi$  and let  $i \in \mathbb{N}$ . Then we have  $\mathcal{B}_0^{\leq i}(\mathcal{T}(e)[i + 1]) = !_{e,i}[\mathcal{N}_{i,e}]$ . Furthermore, for any  $j \in \mathcal{N}_i(e)$ , we have  $\text{im}(b_{\mathcal{T}(e)[i+1]}(!_{e,i}(j))) = \mathcal{K}_{k,j}(\mathcal{T}(e)[i])$ .

# Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$ : $!_{f,1}[\mathcal{N}_1(f)]$



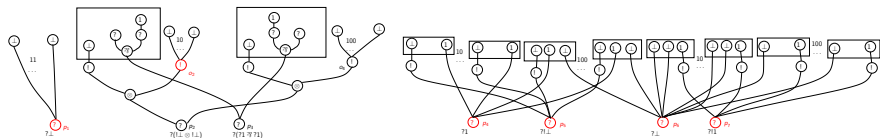
We have  $!_{f,1}[\mathcal{N}_1(f)] = \{o_2, o_4\}$ .

Recovering boxes of depth 1 at depth 0 of  $\mathcal{T}(f)[2]$ :  
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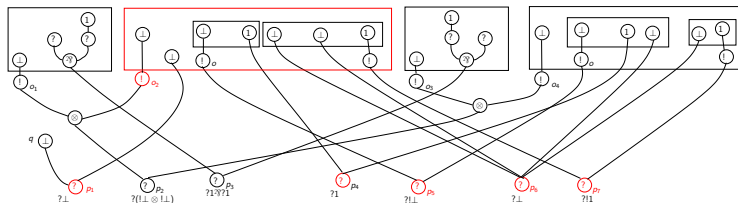
# Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$ : $\mathcal{K}_{10,1}(\mathcal{T}(f)[1])$



We have  $\mathcal{K}_{10,1}(\mathcal{T}(f)[1]) = \{p_1, p_4, p_5, p_6, p_7, o_2\}$

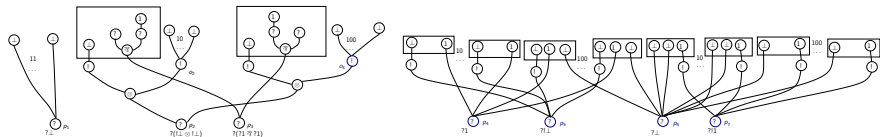


Recovering boxes of depth 1 at depth 0 of  $\mathcal{T}(f)[2]$ :  
 $\text{im}(b_{\mathcal{T}(f)[2]})(o_2)$



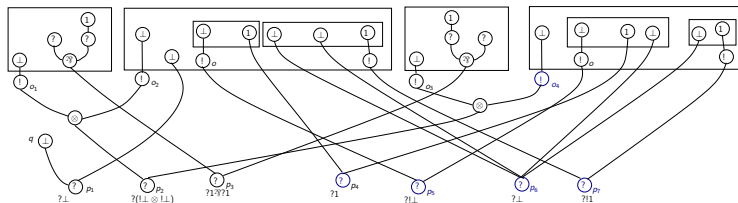
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Recovering boxes of depth 1 at depth 0 of  $\mathcal{T}(f)[2]$ :  
 $\mathcal{K}_{10,2}(\mathcal{T}(f)[1])$



We have  $\mathcal{K}_{10,2}(\mathcal{T}(f)[1]) = \{p_4, p_5, p_6, p_7, o_4\}$

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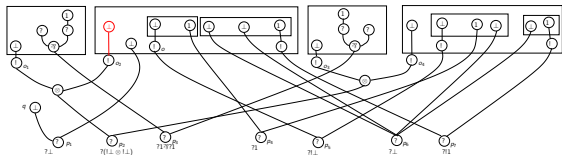
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## From $\mathcal{T}(e)[i]$ to $\mathcal{T}(e)[i + 1]$ (3)

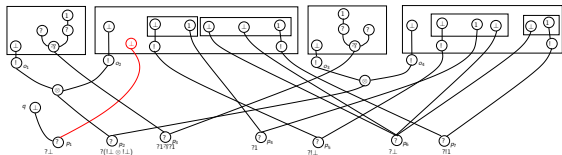
**Proposition.** Let  $\pi$  be a MELL proof-net. Let  $k > \text{Card}(\mathcal{B}(\pi))$ ,  $\text{cosize}(\pi)$ . Let  $e$  be a  $k$ -injective pseudo-experiment of  $\pi$ . Let  $i \in \mathbb{N}$ . Let  $j_0 \in \mathcal{N}_i(e)$ . We set  $\mathcal{T}_{j_0} = \mathcal{S}_{\mathcal{T}(e)[i]}^k(\mathcal{K}_{k,j_0}(\mathcal{T}(e)[i]))$ . Let  $U \in \mathcal{T}_{j_0}$ . Let  $(m_j)_{j \in \mathbb{N}} \in \{0, \dots, k-1\}^{\mathbb{N}}$  such that  $\text{Card}(\{T' \in \mathcal{T}_{j_0}; T' \equiv U\}) = \sum_{j \in \mathbb{N}} m_j \cdot k^j$ . Then we have

$$m_{j_0} = \text{Card} \left( \{T' \in \mathcal{C}^k(\mathcal{B}_{\mathcal{T}(e)[i+1]}(!_{e,i}(j_0))); \mathcal{I}_{e,i,o}(T', \bar{U}) \neq \emptyset\} \right)$$

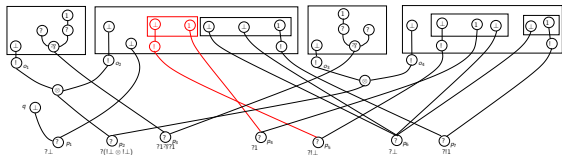
Recovering boxes of depth 1 at depth 0 of  $\mathcal{T}(f)[2]$ :  
 $\mathcal{C}^{10}(B_{\mathcal{T}(f)[2]}(!_{e,i}(1)))$



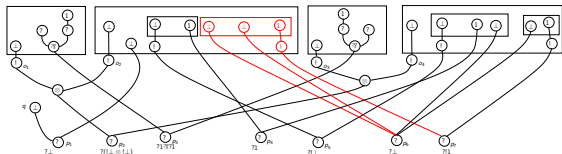
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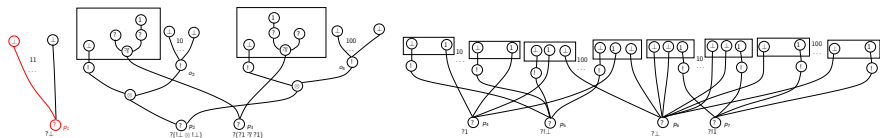


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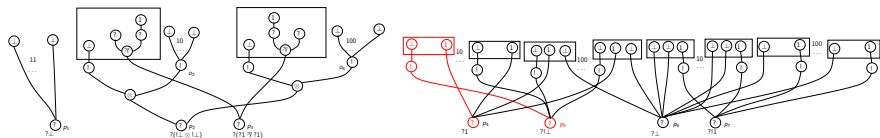


# Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$ : $\mathcal{T}_1$

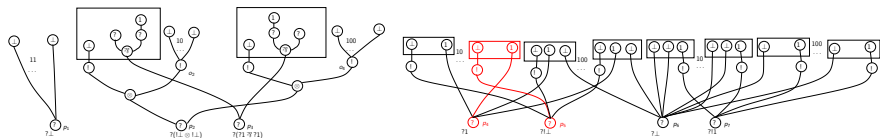




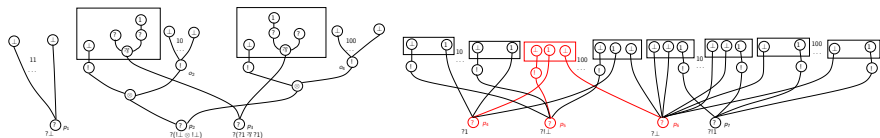
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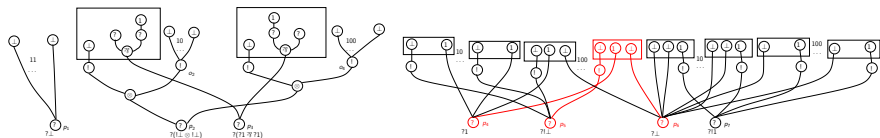
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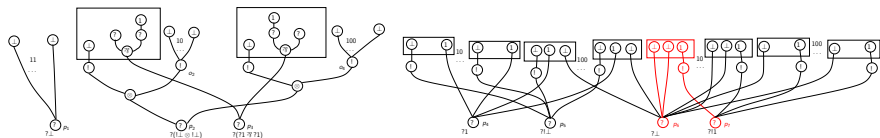
# Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$ : $\mathcal{T}_1$



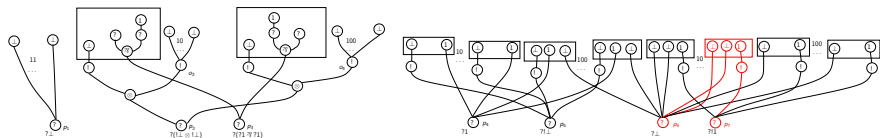
# Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$ : $\mathcal{T}_1$



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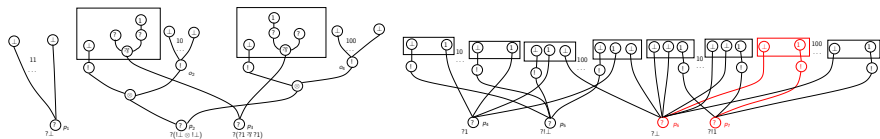


# Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$ : $\mathcal{T}_1$

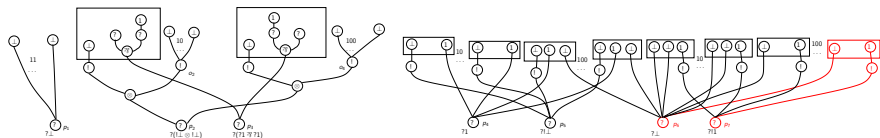




# Recovering boxes of depth 1 at depth 0 of $\mathcal{T}(f)[2]$ : $\mathcal{T}_1$



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# Confluence

Let  $\pi_1$  and  $\pi_2$  cut-free s.t.  $\pi \rightarrow^* \pi_1$  and  $\pi \rightarrow^* \pi_2$ . We have  $[[\pi_1]] = [[\pi_2]]$ , hence  $\pi_1 = \pi_2$ .

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  - *non-uniform* coherence semantics (Bucciarelli-Ehrhard, Boudes)
  - finiteness spaces (Ehrhard)
  - weighted sets (Amini-Ehrhard)
- A weak principal typing property holds for normalizable proof-nets.
- The Taylor expansion is injective for cut-free proof-nets; it seems that the same proof should work in presence of cuts.