Semantics for "Enough-Certainty" and Fitting's Embedding of Classical Logic in S4

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Origins

G. Bana - H. Comon POST'12 computer security:

- First-order language based on a "PPT computability" predicate $t_1, \ldots, t_n \rhd t$ to reason about protocol security
- Semantics
  - $t_1, \ldots, t_n, t$ are interpreted as probabilistic polynomial-time (PPT) algorithms accessing an infinite random bit string ω
  - $t_1, \ldots, t_n \rhd t$ intuitively: the output of t is PPT computable from the outputs of $t_1, \ldots, t_n$ except possibly for negligible probability
  - Disjunction intuitively: $t_1, \ldots, t_n \rhd t \vee t_1, \ldots, t_n \rhd t'$ holds if for certain ω's t can be computed (except maybe for negligible probability), for others t'.
  - Negation intuitively: $\neg t_1, \ldots, t_n \rhd t$ holds if on no non-negligible set can t be computed from $t_1, \ldots, t_n$.
  - Existential quantifier is also non-standard, more complex
- FOL deduction is sound with B-C non-Tarskian semantics (Hard!)

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Key: Fitting's Embedding

- Our semantics is a special case of Fitting's embedding of classical logic in S4 (1970) - a reformulation of Cohen's forcing
- Aim of this talk: Identify some general aspects in which Fitting's embedding arises naturally
- Hence bring attention to Fitting's embedding and its possible uses

First Order Language and Structure

- Consider a first-order language:
  - $\mathfrak{f}$: a set of function symbols
  - $\mathfrak{p}$: a set of predicate symbols
  - $\mathcal{X}$: a set of variables
  - $A_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$: atomic formulas built on $(\mathfrak{f},\mathfrak{p},\mathcal{X})$
  - $\Phi_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$: the first order formulas built on $(\mathfrak{f},\mathfrak{p},\mathcal{X})$ with the same set of variables (with $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\exists$, $\forall$, *true*, *false*)
- First-order structure: Domain $\mathcal{D}$ together with interpretation I
  - I assigns functions on $\mathcal{D}$ to function symbols
  - I assigns relations on $\mathcal{D}$ to predicate symbols
- let $\mathcal{V}$ denote the set of valuations of variables in $\mathcal{D}$
  - for $V \in \mathcal{V}$, $V : \mathcal{X} \rightarrow \mathcal{D}$.
- Having fixed a V valuation, for any term t, the interpretation $I_V(t) \in \mathcal{D}$ is defined the usual way.

Tarskian Semantics of First Order Formulas

Let $w = (\mathcal{D}, I)$ be a first-order structure for $(\mathfrak{f}, \mathfrak{p}, \mathcal{X})$.
An atomic formula ϕ looks like $p(t_1, ..., t_n)$ with $p \in \mathfrak{p}$ and t_i terms.

- $V, w \models p(t_1, ..., t_n) \quad \Leftrightarrow \quad (I_V(t_1), ..., I_V(t_n)) \in I(p)$
- $V, w \models \phi_1 \vee \phi_2 \quad \Leftrightarrow \quad V, w \models \phi_1 \quad \vee \quad V, w \models \phi_2$
- $V, w \models \phi_1 \wedge \phi_2 \quad \Leftrightarrow \quad V, w \models \phi_1 \quad \wedge \quad V, w \models \phi_2$
- $V, w \models \neg\phi \quad \Leftrightarrow \quad V, w \not\models \phi$
- $V, w \models \phi \rightarrow \psi \quad \Leftrightarrow \quad (V, w \models \phi \Rightarrow V, w \models \psi)$
- $V, w \models \exists x \phi[x]$
  $\Leftrightarrow \quad \exists V' \in \mathcal{V}.(V'$ differs from V only on $x \quad \wedge \quad V', w \models \phi[x])$
- $V, w \models \forall x \phi[x]$
  $\Leftrightarrow \quad \forall V' \in \mathcal{V}.(V'$ differs from V only on $x \quad \Rightarrow \quad V', w \models \phi[x])$

Possible Worlds

Consider the following situation:

- $(\mathfrak{f}, \mathfrak{p}, \mathcal{X})$ as before
- Fix a single domain $\mathcal{D}$
- $\mathcal{V}$: valuations as before
- W: a set of possible worlds
- I^w: an interpretation of $\mathfrak{f}$ and $\mathfrak{p}$ for each $w \in W$ (fixed on $\mathfrak{f}$ but varying on $\mathfrak{p}$)
- $\models$: Tarskian satisfaction relation on $\mathcal{V} \times W \times \Phi_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$ written as $V, w \models \phi$
- Define $V, W \models_{\mathrm{C}} \phi \Leftrightarrow \forall w \in W.(V, w \models \phi)$
  (C stands for "covering")
- Clearly, first order deduction rules are sound for $\models_{\mathrm{C}}$ because if some deduction is sound for $\models$, then it works for all w hence for $\models_{\mathrm{C}}$.

Sets of Possible Worlds

- $\models_{\mathrm{C}}$ is not Tarski-semantics for W
- $\models_{\mathrm{C}}$ can equivalently be defined recursively the following way (let $S \subseteq W$):
  - If $\phi \in A_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$, then $V, S \models_{\mathrm{C}} \phi$ iff $\forall w \in W.(V, w \models \phi)$
  - $V, S \models_{\mathrm{C}} \phi_1 \vee \phi_2$ iff there are $S_1, S_2 \subseteq W$ such that $S = S_1 \cup S_2$ and $V, S_1 \models_{\mathrm{C}} \phi_1$ and $V, S_2 \models_{\mathrm{C}} \phi_2$
  - $V, S \models_{\mathrm{C}} \neg\phi \;\Leftrightarrow\; \forall S' \subseteq S.\big(V, S' \not\models_{\mathrm{C}} \phi\big)$
  - $V, S \models_{\mathrm{C}} \phi_1 \rightarrow \phi_2$ iff $\forall S' \subseteq S$, $V, S' \models_{\mathrm{C}} \phi_1$ implies $V, S' \models_{\mathrm{C}} \phi_2$
  - $V, S \models_{\mathrm{C}} \exists x \phi[x]$ iff $\exists \mathcal{S} \subseteq 2^W$ such that $S = \bigcup_{S' \in \mathcal{S}} S'$ and for all $S' \in \mathcal{S}$, there is a $V' \in \mathcal{V}$ differing from V only on x such that $V', S' \models_{\mathrm{C}} \phi[x]$
  - The semantics of $\wedge$ and $\forall$ are the Tarskian ones.

Then $V, W \models_{\mathrm{C}} \phi$ is the same as before

S5 Connection

If we put an accessibility relation $\mathcal{R} \subseteq W \times W$ on W such that $w\mathcal{R}w'$ iff $w = w'$, then W becomes an $S5$ Kripke structure. Define $\phi \mapsto \phi^\circ$ as

- For any atomic formula ϕ, let $\phi^\circ \equiv \Box\phi$.
- $(\phi_1 \vee \phi_2)^\circ \equiv \Box(\phi_1^\circ \vee \phi_2^\circ)$
- $(\neg\phi)^\circ \equiv \Box\neg\phi^\circ$
- $(\phi_1 \to \phi_2)^\circ \equiv \Box(\phi_1^\circ \to \phi_2^\circ)$
- $(\exists x\phi)^\circ \equiv \Box\exists x\phi^\circ$
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Hence soundness of first-order deduction for $\models_{\mathrm{C}}$ can be viewed as a consequence of this embedding theorem.

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First-order deduction rules are not sound if

$$[\phi]_V := \{w \in W | V, w \models \phi\} \in \mathcal{L}$$

is not guaranteed for all ϕ and V.

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  - Negligible probability in complexity theory: $(W, \Sigma, P_1, P_2, ...)$, $S \in \Sigma$, $\forall m \in \mathbb{N}.\ \exists i_0 \in \mathbb{N}.\ \forall i > i_0.\ \frac{1}{i^m} > P_i(S)$

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- Let's distinguish significant and insignificant sets:
  - $\mathcal{L} = \mathcal{L}_{\mathrm{s}} \cup \mathcal{L}_{\mathrm{i}}$
  - $\mathcal{L}_{\mathrm{i}}$ is a non-trivial ideal of $\mathcal{L}$:
    - ★ if $S \in \mathcal{L}_{\mathrm{i}}$, and $S' \in \mathcal{L}$, and $S' \subseteq S$, then $S' \in \mathcal{L}_{\mathrm{i}}$;
    - ★ if $S \in \mathcal{L}_{\mathrm{i}}$, and $S' \in \mathcal{L}_{\mathrm{i}}$, then then $S \cup S' \in \mathcal{L}_{\mathrm{i}}$;
    - ★ $\mathcal{L}_{\mathrm{i}} \neq \mathcal{L}$.

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    - if $S \in \mathcal{L}_\mathrm{i}$, and $S' \in \mathcal{L}_\mathrm{i}$, then then $S \cup S' \in \mathcal{L}_\mathrm{i}$;
    - $\mathcal{L}_\mathrm{i} \neq \mathcal{L}$.
- Problems arise from that infinite unions of insignificant sets may be significant.

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- $\Omega \models X = Y \vee U = V$:
  - $X = Y$ holds almost everywhere on one part of Ω
  - $U = V$ holds almost everywhere on the rest of Ω
  - E.g. The result of a coin toss is either Heads or Tails (lending on the edge has 0 probability)
  - E.g. The result of casting a dice is either ≤ 5 or even.
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  - E.g. The coin does not fall on its edge
- Exist: If we roll a homogenous ball, there is an open hemisphere on which it stops with non-zero probability, but there is no surface point on which it stops with non-zero probability.
  - We can cover Ω with significant sets on each of which there is a witness

Let $(\mathcal{D}, W, \mathcal{L}, \mathcal{L}_{\rm s}, \mathfrak{f}, \mathfrak{p}, \mathcal{X}, I^w)$ be as before, define $\models_{\rm CEC}$ on $\mathcal{V} \times \mathcal{L}_{\rm s} \times \Phi_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$

- If $\phi \in A_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$, then $V, S \models_{\rm CEC} \phi$ iff $\exists S' \in \mathcal{L}_{\rm s}.(S \setminus S' \in \mathcal{L}_{\rm i} \ \wedge \ V, S' \models_{\rm C} \phi)$.
- $V, S \models_{\rm CEC} \phi_1 \vee \phi_2 \ \Leftrightarrow \ \exists S_1, S_2 \in \mathcal{L}_{\rm s}.\big(S \setminus (S_1 \cup S_2) \in \mathcal{L}_{\rm i} \ \wedge \ V, S_1 \models_{\rm CEC} \phi_1 \ \wedge \ V, S_2 \models_{\rm CEC} \phi_2\big)$
- $V, S \models_{\rm CEC} \neg\phi \ \Leftrightarrow \ \forall S' \in \mathcal{L}_{\rm s}.\big(S' \subseteq S \ \Rightarrow \ V, S' \not\models_{\rm CEC} \phi\big)$
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- $V, S \models_{\rm CEC} \exists x\phi[x]$
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- The semantics of $\wedge$ and $\forall$ are the usual ones.

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Two issues with $\exists$...

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- $V, S \models_{\rm CEC} \exists x\phi[x] \ \Leftrightarrow \forall S' \in \mathcal{L}_{\rm s}.\Big(S' \subseteq S \Rightarrow$
  $\exists \mathcal{S} \subseteq \mathcal{L}_{\rm s}.\ \exists S''' \in \mathcal{L}_{\rm i}.\Big(S' \setminus \bigcup_{S'' \in \mathcal{S}} S'' \subseteq S''' \ \wedge \ \forall S'' \in \mathcal{S}.(\exists V' \in \mathcal{V}$
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- The semantics of $\wedge$ and $\forall$ are the usual ones.

Covering Enough-Certainty Semantics

Let $(\mathcal{D}, W, \mathcal{L}, \mathcal{L}_{\rm s}, \mathfrak{f}, \mathfrak{p}, \mathcal{X}, I^w)$ be as before, define $\models_{\rm CEC}$ on $\mathcal{V} \times \mathcal{L}_{\rm s} \times \Phi_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$

- If $\phi \in A_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$, then $V, S \models_{\rm CEC} \phi$ iff $\exists S' \in \mathcal{L}_{\rm s}.(S \setminus S' \in \mathcal{L}_{\rm i} \ \wedge \ V, S' \models_{\rm C} \phi)$.
- $V, S \models_{\rm CEC} \phi_1 \vee \phi_2 \ \Leftrightarrow \ \exists S_1, S_2 \in \mathcal{L}_{\rm s}.\big(S \setminus (S_1 \cup S_2) \in \mathcal{L}_{\rm i} \ \wedge \ V, S_1 \models_{\rm CEC} \phi_1 \ \wedge \ V, S_2 \models_{\rm CEC} \phi_2\big)$
- $V, S \models_{\rm CEC} \neg\phi \ \Leftrightarrow \ \forall S' \in \mathcal{L}_{\rm s}.\big(S' \subseteq S \ \Rightarrow \ V, S' \not\models_{\rm CEC} \phi\big)$
- $V, S \models_{\rm CEC} \phi_1 \rightarrow \phi_2 \ \Leftrightarrow \ \forall S' \in \mathcal{L}_{\rm s}.\big(S' \subseteq S \ \wedge \ V, S' \models_{\rm CEC} \phi_1 \Rightarrow V, S' \models_{\rm CEC} \phi_2\big)$
- $V, S \models_{\rm CEC} \exists x \phi[x] \ \Leftrightarrow \forall S' \in \mathcal{L}_{\rm s}.\Big(S' \subseteq S \Rightarrow \exists \mathcal{S} \subseteq \mathcal{L}_{\rm s}.\ \exists S''' \in \mathcal{L}_{\rm i}.\Big(S' \setminus \bigcup_{S'' \in \mathcal{S}} S'' \subseteq S''' \ \wedge \ \forall S'' \in \mathcal{S}.(\exists V' \in \mathcal{V} \text{ differing from } V \text{ only on } x.\ (V', S'' \models_{\rm CEC} \phi[x]))\Big)\Big)$
- The semantics of $\wedge$ and $\forall$ are the usual ones.

First-order deduction is not sound.

Even if $[\phi]_V := \{w \in W | V, w \models \phi\} \in \mathcal{L}$, witnesses might exist only on insignificant sets, so the soundness proof before does not work.

Fitting Embedding

For any first-order formula ϕ, consider the transformation $\phi \to \phi^*$,

- For any atomic formula ϕ, let $\phi^* \equiv \Box\Diamond\phi$.
- $(\phi_1 \vee \phi_2)^* \equiv \Box\Diamond(\phi_1^* \vee \phi_2^*)$
- $(\neg\phi)^* \equiv \Box\neg\phi^*$
- $(\phi_1 \to \phi_2)^* \equiv \Box(\phi_1^* \to \phi_2^*)$
- $(\exists x\phi)^* \equiv \Box\Diamond\exists x\phi^*$
- $(\phi_1 \wedge \phi_2)^* \equiv (\phi_1^* \wedge \phi_2^*)$
- $(\forall x\phi)^* \equiv \forall x\phi^*$

We assume Barcan formulas: $\forall x\Box\phi \leftrightarrow \Box\forall x\phi$ (single domain)
$\Box\Diamond$ can be written everywhere, but it simplifies to the above

Theorem (Fitting's Embedding, 1970)

Any formula ϕ is derivable in first-order logic if and only if ϕ^* it is derivable in S4 with the Barcan formulas.

S4 structure of significant sets

Take $(\mathcal{D}, W, \mathcal{L}, \mathcal{L}_{\mathrm{s}}, \mathfrak{f}, \mathfrak{p}, \mathcal{X})$ as before. Defining $\mathcal{R} \subseteq \mathcal{L}_{\mathrm{s}} \times \mathcal{L}_{\mathrm{s}}$, $S\mathcal{R}S' \Leftrightarrow S' \subseteq S$, we have a transitive, reflexive accessibility relation, hence with $\subseteq$, $\mathcal{L}_{\mathrm{s}}$ is a S4 Kripke-structure.

Suppose for each $S \in \mathcal{L}_{\mathrm{s}}$, $\mathcal{I}^S$ interprets $\mathfrak{f}$ and $\mathfrak{p}$ on $\mathcal{D}$

Fitting Twisted Enough-Certainty Semantics

We can define the relation $\models_{\mathrm{FEC}}$ on $\mathcal{V} \times \mathcal{L}_{\mathrm{s}} \times \Phi_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$ in the following way:

- $p(t_1, ..., t_n) \in A_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$, then $V, S \models_{\mathrm{FEC}} \phi$
  $\Leftrightarrow \forall S' \in \mathcal{L}_{\mathrm{s}}.\ \Big(S' \subseteq S \Rightarrow \exists S'' \in \mathcal{L}_{\mathrm{s}}.\ (S'' \subseteq S' \wedge (\mathcal{I}_V^{S''}(t_1), ..., \mathcal{I}_V^{S''}(t_n)) \in \mathcal{I}^{S''}(p))\Big)$
- $V, S \models_{\mathrm{FEC}} \phi_1 \vee \phi_2 \ \Leftrightarrow\ \forall S' \in \mathcal{L}_{\mathrm{s}}.\ \Big(S' \subseteq S\ \Rightarrow\ \exists S'' \in \mathcal{L}_{\mathrm{s}}.\ (S'' \subseteq S'$
  $\wedge\ (V, S'' \models_{\mathrm{FEC}} \phi_1\ \vee\ V, S'' \models_{\mathrm{FEC}} \phi_2))\Big)$
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- $V, S \models_{\mathrm{FEC}} \phi \rightarrow \psi\ \Leftrightarrow\ \forall S' \in \mathcal{L}_{\mathrm{s}}.\ \Big(S' \subseteq S \wedge V, S' \models_{\mathrm{FEC}} \phi \Rightarrow V, S' \models_{\mathrm{FEC}} \psi\Big)$
- $V, S \models_{\mathrm{FEC}} \exists x\phi[x]\ \Leftrightarrow\ \forall S' \in \mathcal{L}_{\mathrm{s}}.\ \Big(S' \subseteq S$
  $\Rightarrow\ \exists S'' \in \mathcal{L}_{\mathrm{s}}.\ (S'' \subseteq S' \wedge (\exists V' \in \mathcal{V}$
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$V, S \models_{\mathrm{FEC}} \phi \Leftrightarrow V, S \models^{K} \phi^*$

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$V, S \models_{\mathrm{FEC}} \phi \Leftrightarrow V, S \models^K \phi^*$

Since by Fitting, first-order deduction is sound for $V, S \models^K \phi^*$, it is also sound for $V, S \models_{\mathrm{FEC}} \phi$.

Compare

- $V, S \models_{\mathrm{CEC}} \phi_1 \vee \phi_2$
  $\Leftrightarrow\ \exists S_1, S_2 \in \mathcal{L}_{\mathrm{s}}.(S \setminus (S_1 \cup S_2) \in \mathcal{L}_{\mathrm{i}}\ \wedge\ \ V, S_1 \models_{\mathrm{CEC}} \phi_1\ \ \wedge\ \ V, S_2 \models_{\mathrm{CEC}} \phi_2)$

- $V, S \models_{\mathrm{FEC}} \phi_1 \vee \phi_2\ \Leftrightarrow\ \forall S' \in \mathcal{L}_{\mathrm{s}}.\ \Big(S' \subseteq S\ \Rightarrow\ \exists S'' \in \mathcal{L}_{\mathrm{s}}.\ (S'' \subseteq S'$
  $\wedge\ (V, S'' \models_{\mathrm{FEC}} \phi_1\ \vee\ V, S'' \models_{\mathrm{FEC}} \phi_2))\Big)$

Covering Is Sometimes Fitting

Theorem

Let $(\mathcal{D}, W, \mathcal{L}, \mathcal{L}_{\mathrm{s}}, \mathfrak{f}, \mathfrak{p}, \mathcal{X}, I^w)$ *be as before. Let* $\mathtt{card}(\mathcal{D})$ *denote the cardinality of* $\mathcal{D}$*. If* $[\phi]_V := \{w : V, w \models \phi\} \in \mathcal{L}$ *for* $\phi \in A_{(\mathfrak{f},\mathfrak{p},\mathcal{X})}$ *and valuation* V *of variables in* ϕ*, and if* $\mathcal{L}$ *is closed under taking unions of as many of its subsets as the cardinality* $\mathtt{card}(\mathcal{D})$*, then covering enough-certainty semantics is a special case of Fitting twisted enough-certainty semantics.*

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Conditions of the theorem are satisfied: can use CEC

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Conditions of the theorem are not satisfied: use FEC - which is what we did for computability.

Applications

- Computer security: properties allowed to fail on negligible sets

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- Computer security: properties allowed to fail on negligible sets
- Conditional logic - default logic
  - Conditional implication $\mapsto$ for something to hold "typically"
  - Various ways in the literature to give a semantics for $\mapsto$
  - We can define a semantics that satisfies usual desired properties
    - KLM axioms, satisfiability of Lottery and Crooked Lottery paradoxes
  - Our $\mapsto$ turns into material implication when atypical sets are insignificant as a result of the Fitting twist

Computability Predicate 1

What should be the computational semantics of $t_1, ..., t_n \triangleright t$?

- Possible worlds: $W = \{(1^\eta, \omega) | \omega \text{ infinite random bit string}\}$. For each fixed η, we have a probability distribution P_η on W.
- $\mathcal{D}$: set of probabilistic polynomial-time algorithms with input $(1^\eta, \omega)$
- Significant sets: Non-negligible measurable subsets of W

Computability Predicate 2

$V, S \overset{c}{\models} t_1, ..., t_n \rhd t$?

- First try: There is a PPT A such that $A(t_1(1^\eta, \omega), ..., t_n(1^\eta, \omega), \omega) = t(1^\eta, \omega)$ for almost all $(1^\eta, \omega) \in W$

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  - No: Has bad properties: $t_1, ..., t_n \rhd t \wedge t_1, ..., t_n \rhd t'$ would not imply $t_1, ..., t_n \rhd (t, t')$ because sets on which t is computable and on which t' is computable might be disjoint

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- Third try: There is a set of non-negligible sets $\mathcal{S}$ such that $\bigcup_{S\in\mathcal{S}} S$ almost covers W and for each $S \in \mathcal{S}$, there is a PPT A_S such that $A_S(t_1(1^\eta,\omega), \ldots, t_n(1^\eta,\omega),\omega) = t(1^\eta,\omega)$ for all $(1^\eta,\omega) \in S$.

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- Fourth try: For any non-negligible set S, there is a set of non-negligible sets $\mathcal{S}$ such that $\bigcup_{S'\in\mathcal{S}} S'$ almost covers S and for each $S' \in \mathcal{S}$, there is a PPT $A_{S'}$ such that $A_{S'}(t_1(1^\eta,\omega), ..., t_n(1^\eta,\omega),\omega) = t(1^\eta,\omega)$ for all $(1^\eta,\omega) \in S'$.

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- Simplify: For any non-negligible set S, there is an S' non-negligible subset of S and a PPT $A_{S'}$ such that $A_{S'}(t_1(1^\eta, \omega), \ldots, t_n(1^\eta, \omega), \omega) = t(1^\eta, \omega)$ for all $(1^\eta, \omega) \in S$.

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For future:

- We are exploring more the applicability for conditional logic
- Other applications?