A Sequent Calculus for a Modal Logic on Finite Data Trees

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XML and XPath

- An XML tree :

$\langle$ catalogue $d = "\_" \rangle$

$\langle$ category $d =$ "SF" $\rangle$ $\qquad$ $\langle$ category $d =$ "Poetry" $\rangle$

$\langle$ author $d =$ "SF" $\rangle$ $\qquad$ $\langle$ author $d =$ "Poetry" $\rangle$

- A request : $\Box($ category $\supset \Diamond_{=}$ author $)$

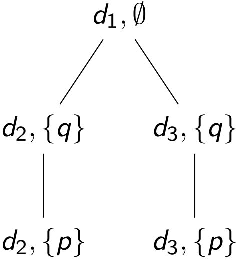
Validity problem for modal data logic

- Modal data logic : addressed by model theory and combinatorial algorithms (e.g. Figueira, 2010)
- Want to study it with proof theory
  - $\rightarrow$ witness for validity of a formula (proof tree) and a counter-example otherwise
  - $\rightarrow$ amenable to reasonable implementations avoiding worst-case complexity
  - $\rightarrow$ same complexity as combinatorial algorithms

Data tree

A *data tree* $\mathcal{A} = (A, \delta, \mathcal{V})$ is a tree $A = (T, <)$ where :

- T the set of nodes of A,
- $<$ the usual descendant relation on trees,
- $\delta \colon T \longrightarrow \mathbb{D}$ a function which associates a datum from a countable set $\mathbb{D}$ to a node of A,
- $\mathcal{V} \colon T \longrightarrow \mathcal{P}(\mathbb{A})$ a valuation which associates a set of atomic propositions to every node of A.



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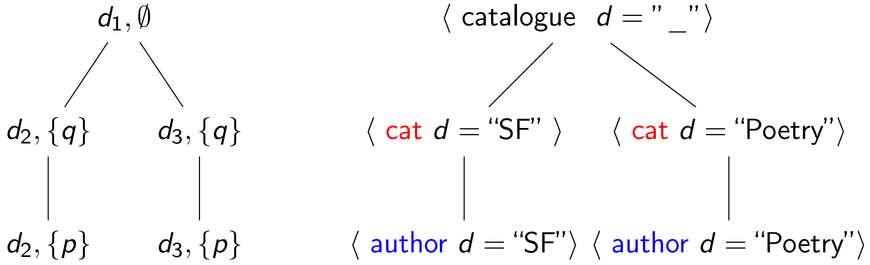

Modal Data Logic (DataGL)

Syntax

$\phi ::= a \mid \bot \mid \phi \supset \phi \mid \Box_{=}\phi \mid \Box_{\neq}\phi$,
where $a \in \mathbb{A}$ the set of atomic propositions.

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Semantic

Let $\mathcal{A} = (A, <, \delta, \mathcal{V})$ be a *finite data tree*.

- $t \models a$ iff $a \in \mathcal{V}(w)$.
- $t \not\models \bot$
- $t \models \phi \supset \psi$ iff if $t \models \phi$, then $t \models \psi$.
- $t \models \Box_{=}\phi$ iff $\forall t_1 < t$ s.t. $\delta(t_1) = \delta(t)$, then $t_1 \models \phi$.
- $t \models \Box_{\neq}\psi$ iff $\forall t_1 < t$ s.t. $\delta(t_1) \neq \delta(t)$, then $t_1 \models \psi$.

Definition

Two relations $R^{=}$ and $R^{\neq}$:

$$\begin{cases} tR^{=}t_1 & \textit{iff} \quad t_1 < t \textit{ and } \delta(t_1) = \delta(t), \\ tR^{\neq}t_1 & \textit{iff} \quad t_1 < t \textit{ and } \delta(t_1) \neq \delta(t). \end{cases}$$

DataGL : a bimodal logic

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- $R^{=}$ transitive and irreflexive,
- $R^{\neq}$ irreflexive,
  and if $t_1 R^{\neq} t_2$ and $t_2 R^{\neq} t_3$, then $t_1 R^{\neq} t_3$ or $t_1 R^{=} t_3$.

DataGL : an example

Example

- $\Box_{=} p \supset \Box_{=} \Box_{=} p$ *is valid,*
- *but* $\Box_{\neq} p \supset \Box_{\neq} \Box_{\neq} p$ *is not.*

$$
\begin{tikzcd}
d_1, \emptyset \models \Box_{\neq} p \wedge \neg \Box_{\neq} \Box_{\neq} p \arrow[d, dash, "R^{\neq}"] \arrow[dd, dash, dashed, bend right, "R^{=}"'] \\
d_2, \{p\} \models \neg \Box_{\neq} p \arrow[d, dash, "R^{\neq}"] \\
d_1, \emptyset \models \neg p
\end{tikzcd}
$$

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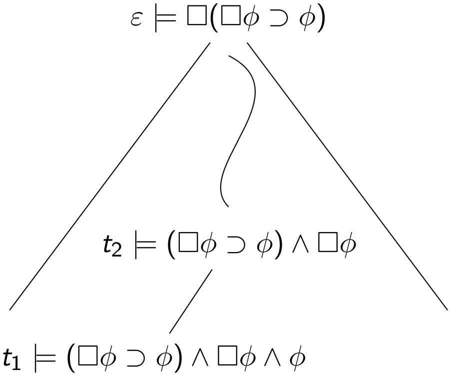

Avron's sequent calculus (1984)

Complete sequent calculus for GL
extending a standard propositional calculus with :

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- $\Box\phi$ is here to integrate axiom (L).

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- $\Box_=$ has the same behavior as $\Box$,
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$$\frac{\Box_= \Gamma^=, \Gamma^=, \Box_{\neq} \Gamma^{\neq}, \Box_= \phi \Rightarrow \phi}{\Box_= \Gamma^=, \Box_{\neq} \Gamma^{\neq} \Rightarrow \Box_= \phi} \ \Box_=$$

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$$\varepsilon \models \Box_=\Gamma^= \wedge \Box_{\neq}\Gamma^{\neq}$$

$$\Big|\ R^=$$

$$t \models \Box_=\Gamma^= \wedge \Gamma^= \wedge \Box_{\neq}\Gamma^{\neq} \wedge \Box_=\phi$$

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  - $\rightarrow$ Follow $R^{\neq}$ n times $\Rightarrow$ n cases to consider.
  - $\rightarrow$ Furthermore, need to remember the previous hypotheses.
- Solution : Use an history $\mathcal{H}$ which remembers the hypotheses previously encountered.

History

The history $\mathcal{H}$ is a finite multi-set of sets of formulæ.
We will denote it $\mathcal{H} = H_0; \ldots; H_n$.

Rule for the modality $\Box_{\neq}$

- The two-premises case :

$$\dfrac{\dfrac{\ldots}{\ldots}}{\Box_{=} H_0^{=}, \Box_{\neq} H_0^{\neq}; \Box_{=} \Gamma^{=}, \Box_{\neq} \Gamma^{\neq} \Rightarrow \Box_{\neq} \psi}\ \Box_{\neq}$$

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$$\Big| \ R^{\neq}$$

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 & t \models \Box_= \Gamma^= \wedge \Box_{\neq} \Gamma^{\neq} \arrow[dl, dash, "R^{\neq}"] \\
t_1 \models H_0^{\neq} \wedge \Gamma^{\neq} &
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t_1 \models H_0^{\neq} \wedge \Gamma^{\neq} & & t_2 \models \Box_= H_0^= \wedge H_0^= \wedge \Box_{\neq} H_0^{\neq} \wedge \Gamma^{\neq}
\end{tikzcd}$$

Example of a proof search

$$\dfrac{}{\;;\;\Rightarrow \Box_{\neq}\psi \supset \Box_{\neq}\Box_{\neq}\psi}$$

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$$\dfrac{\overline{\quad ; \Box_{\neq}\psi \Rightarrow \Box_{\neq}\Box_{\neq}\psi \quad}}{; \ \Rightarrow \Box_{\neq}\psi \supset \Box_{\neq}\Box_{\neq}\psi} \ R \supset$$

Example of a proof search

$$\dfrac{\dfrac{\color{blue}{\Box_{\neq}\psi, \Box_{\neq}\Box_{\neq}\psi}; \psi \Rightarrow \color{red}{\Box_{\neq}\psi}}{; \Box_{\neq}\psi \Rightarrow \Box_{\neq}\Box_{\neq}\psi}\ \Box_{\neq}}{;\ \Rightarrow \Box_{\neq}\psi \supset \Box_{\neq}\Box_{\neq}\psi}\ R\supset$$

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$$\dfrac{\dfrac{\dfrac{\overline{\Box_{\neq}\psi, \Box_{\neq}\Box_{\neq}\psi; \Box_{\neq}\psi; \psi, \Box_{\neq}\psi \Rightarrow \psi} \qquad \overline{}}{\Box_{\neq}\psi, \Box_{\neq}\Box_{\neq}\psi; \psi \Rightarrow \Box_{\neq}\psi}\ \Box}{; \Box_{\neq}\psi \Rightarrow \Box_{\neq}\Box_{\neq}\psi}\ \Box_{\neq}}{;\ \Rightarrow \Box_{\neq}\psi \supset \Box_{\neq}\Box_{\neq}\psi}\ R\supset$$

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  \dfrac{\text{Failure}}{\Box_{\neq}\psi; \Box_{\neq}\psi, \Box_{\neq}\Box_{\neq}\psi \Rightarrow \psi}
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  }{; \;\Rightarrow \Box_{\neq}\psi \supset \Box_{\neq}\Box_{\neq}\psi}\,R\supset
}\,\Box
$$

Rule for $\Box_{\neq}$

$$\dfrac{\mathcal{H}; \Box_{=}\Gamma^{=}, \Box_{\neq}\Gamma^{\neq}, \Box_{\neq}\psi; H_i^{\neq}, \Gamma^{\neq} \Rightarrow \psi \qquad (\mathcal{H} \setminus H_i; \Box_{=}\Gamma^{=}, \Box_{\neq}\Gamma^{\neq}, \Box_{\neq}\psi; \Box_{=}H_i^{=}, H_i^{=}, \Box_{\neq}H_i^{\neq}, H_{j_{j\neq i}}^{\neq}, \Gamma^{\neq} \Rightarrow \psi)_i}{\mathcal{H}; \Box_{=}\Gamma^{=}, \Box_{\neq}\Gamma^{\neq} \Rightarrow \Box_{\neq}\psi}\ \Box_{\neq}$$

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- Extraction of the information depends on the case.

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- n premises $=$ n cases to consider, but 2 kind of cases :
  - $\rightarrow$ First premise : datum of the arrival node different from the one of all other nodes.
  - $\rightarrow$ Other premises : datum of the arrival node is the same than the one of a previous node.
- Extraction of the information depends on the case.

Adequation of the calculus

Soundness theorem

The sequent calculus is sound.

Completeness theorem

The sequent calculus is complete.

Small proof property

Let ϕ a provable formula.
We can construct a proof tree of polynomial depth.

Properties of our calculus

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Upper complexity bound

The validity problem is PSPACE-easy.

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Small model property

Let ϕ be a non-provable formula.
We can build a counter-model of polynomial depth.

Lower complexity bound

Chagrov & Rybakov, 2002

The validity problem in GL is PSPACE-hard.

Chagrov & Rybakov, 2002

The validity problem in GL is PSPACE-hard.

Theorem

The validity problem in DataGL is PSPACE-hard.

Lower complexity bound

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Theorem

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Corollary

The validity problem in DataGL is PSPACE-complete.

Conclusion and prospects

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- Prospects :
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  - Extension of the calculus to larger fragments of XPath.

Questions

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