Easy to Win, Hard to Master:
Optimal Strategies in Parity Games with Costs

Joint work with Martin Zimmermann

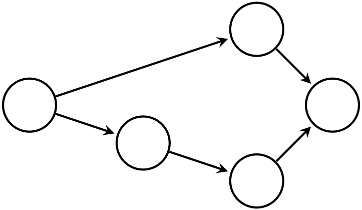
Alexander Weinert

Saarland University

August 31st, 2016

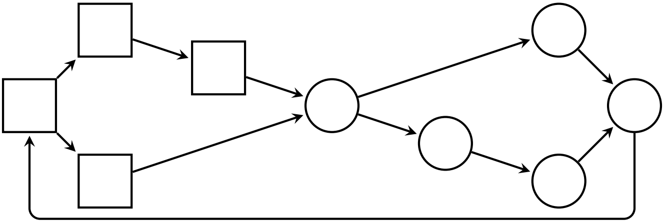
CSL 2016 - Marseille, France

Parity Games



Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Parity Games



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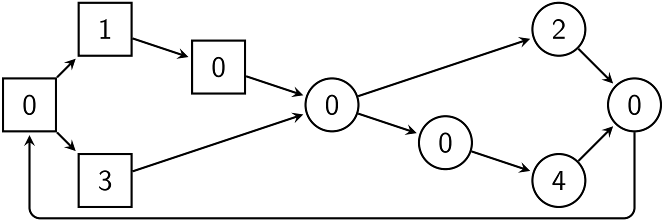

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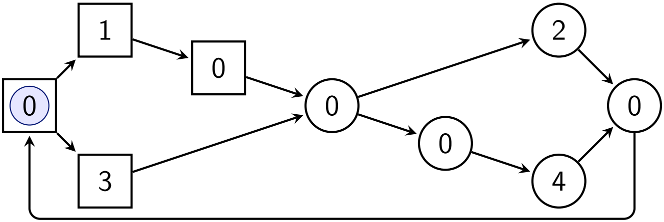

0

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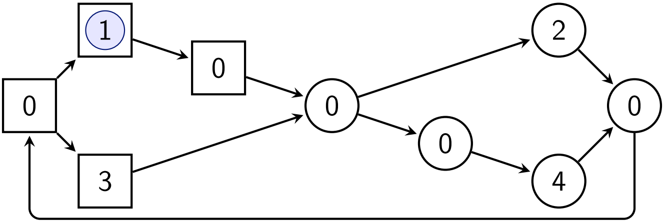
Parity Games



$0 \longrightarrow 1$

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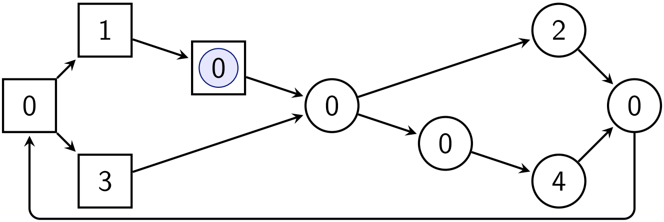
Parity Games



$0 \longrightarrow 1 \longrightarrow 0$

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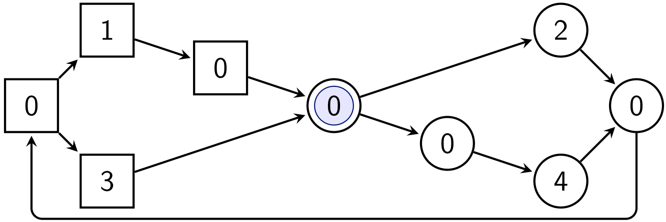

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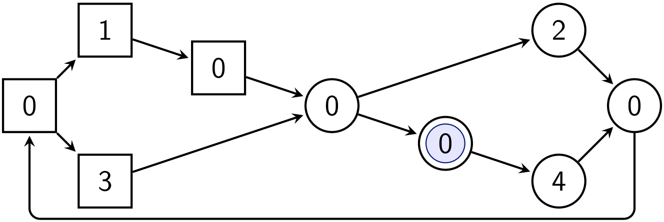

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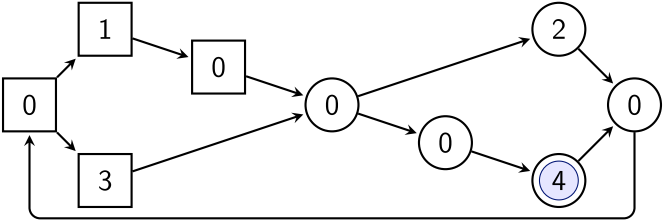

$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4$

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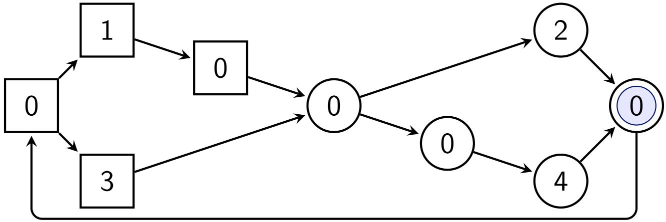

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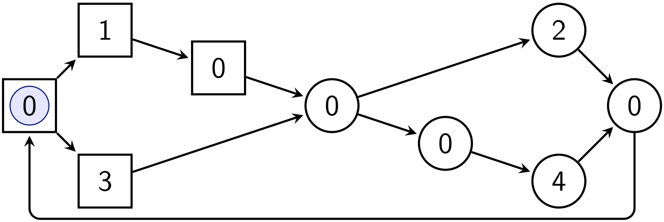

$$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4 \longrightarrow 0$$

$$0$$

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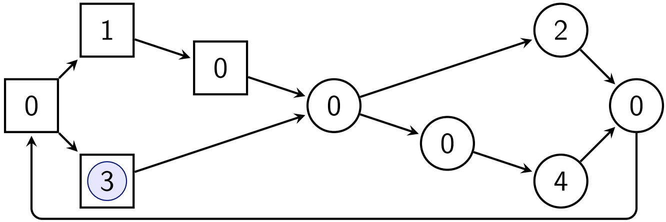

$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4 \longrightarrow 0$

$0 \longrightarrow 3$

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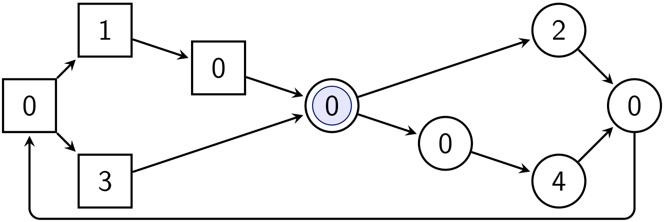

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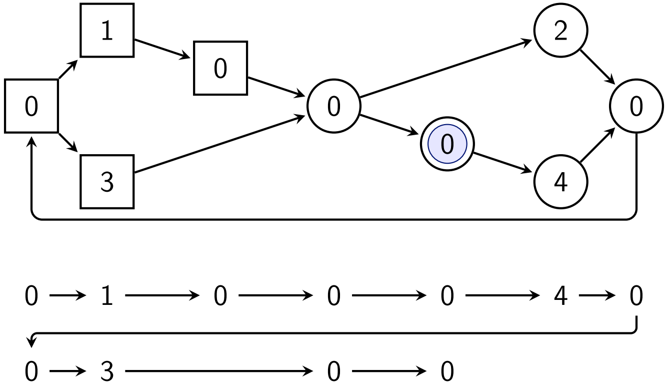

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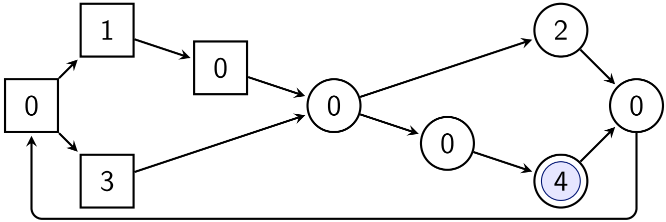

$0 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 4 \rightarrow 0$

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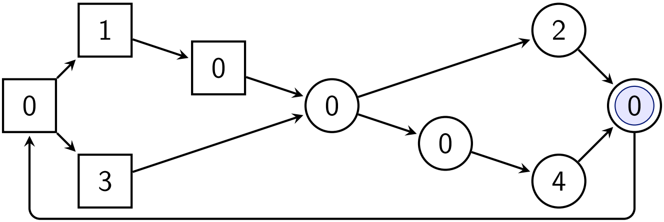

$0 \to 1 \to 0 \to 0 \to 0 \to 4 \to 0$

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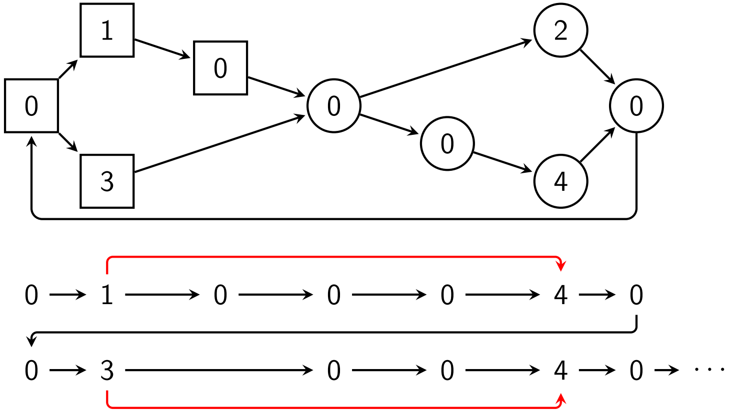

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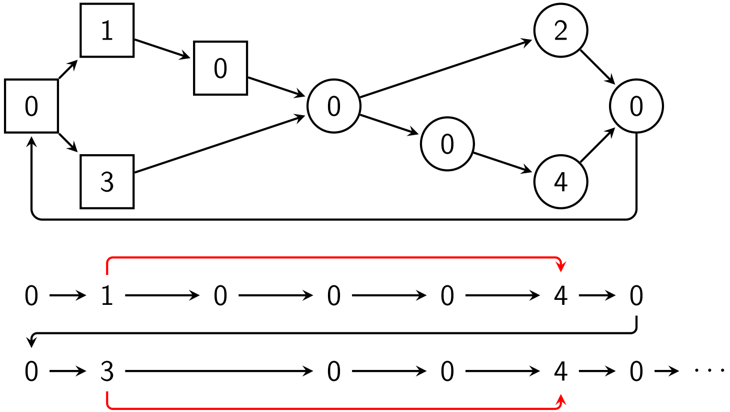
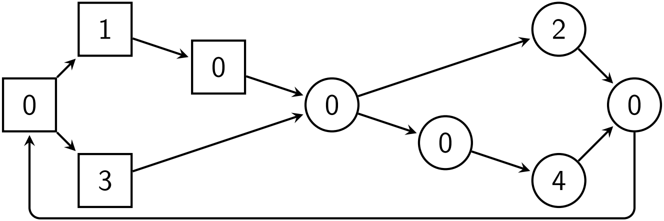


Deciding winner in $\mathrm{UP} \cap \mathrm{co\text{-}UP}$

Positional Strategies

Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Finitary Parity Games



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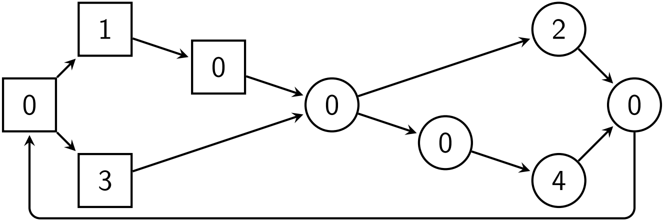

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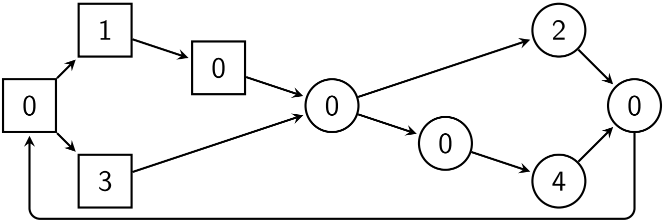

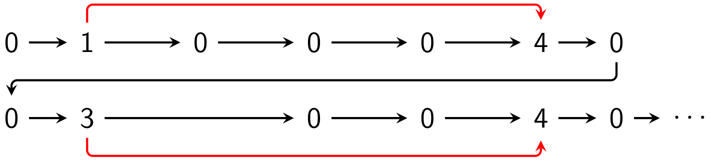

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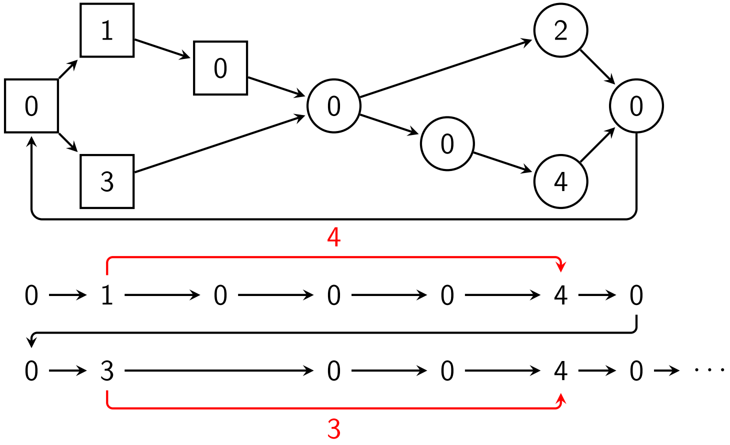

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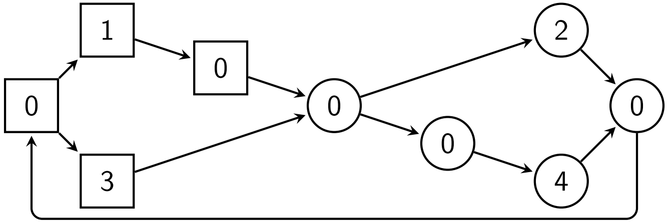

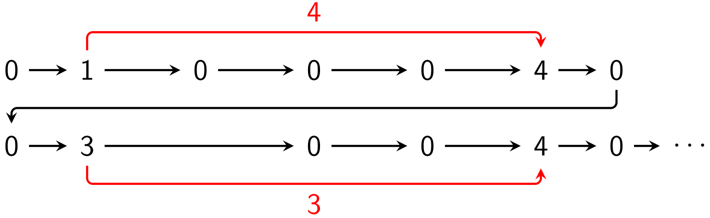

Goal for Player 0: Bound response times

Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Decision Problem

Theorem (Chatterjee et al., Finitary Winning, 2009)

The following decision problem is in $\textsc{PTime}$*:*

Input: *Finitary parity game* $\mathcal{G} = (\mathcal{A}, \mathrm{FinParity}(\Omega))$

Question: *Does there exist a strategy* σ *with* $\mathrm{Cst}(\sigma) < \infty$*?*

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Theorem

The following decision problem is $\textsc{PSpace}$*-complete:*

Input: *Finitary parity game* $\mathcal{G} = (\mathcal{A}, \mathrm{FinParity}(\Omega))$*,*
bound $b \in \mathbb{N}$

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Introduction

Introduction ✓

Introduction ✓

↓

Complexity

in PSPACE

Introduction ✓

↓

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in PSpace

PSpace-hard

Introduction ✓

Complexity

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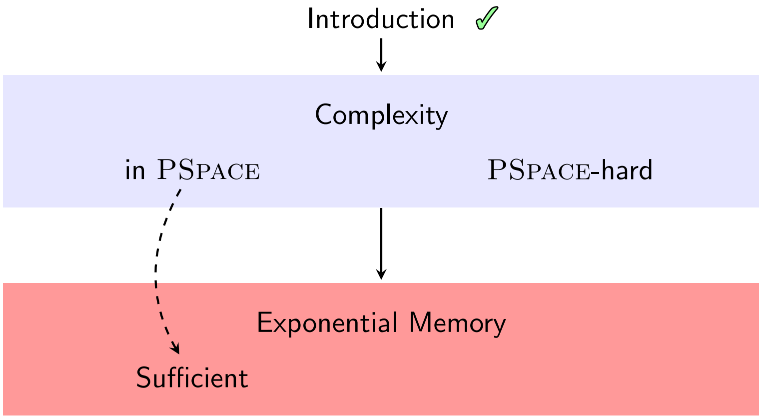
PSpace-hard

Exponential Memory

Sufficient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Necessary

From Finitary Parity to Parity

Given: Finitary parity game $\mathcal{G} = (\mathcal{A}, \mathrm{FinParity}(\Omega))$, bound $b \in \mathbb{N}$.

Lemma

Deciding if Player 0 *has strategy* σ *with* $\mathrm{Cst}(\sigma) \leq b$ *is in* $\mathrm{PS}{\small \mathrm{PACE}}$*.*

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$\Rightarrow$ Problem is in $\mathrm{APT}\textsc{ime}$

(Chandra et al., Alternation, 1981)

$\Rightarrow$ Problem is in $\mathrm{PS}\textsc{pace}$

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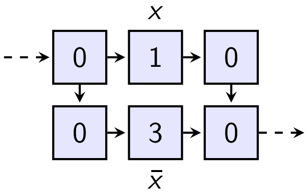

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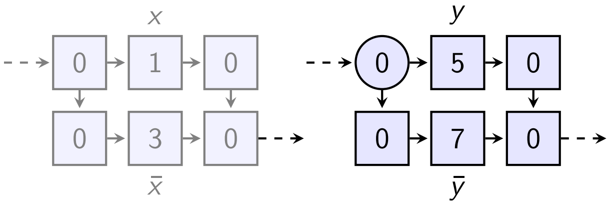

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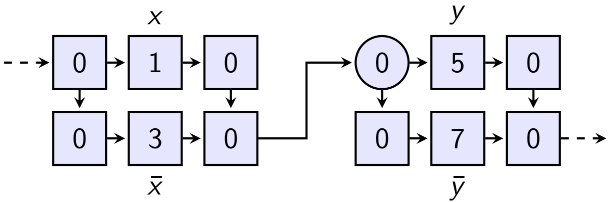

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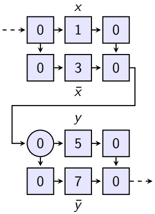

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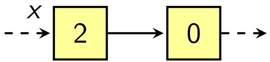

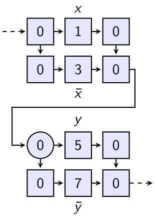

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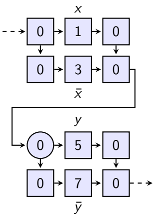

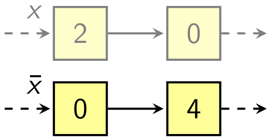

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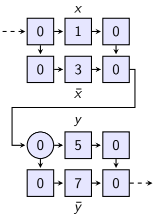

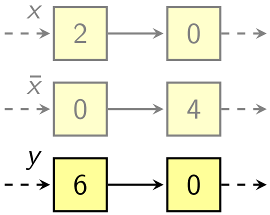

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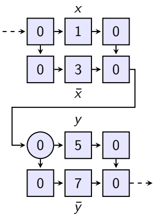

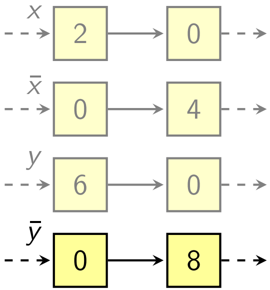

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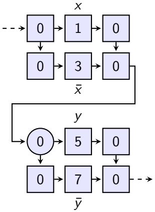

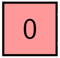

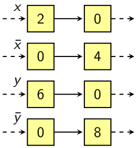

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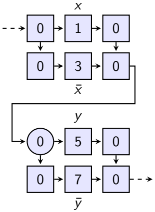

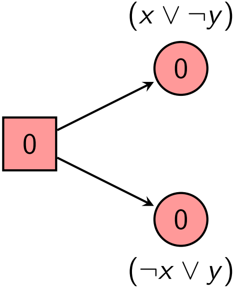

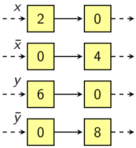

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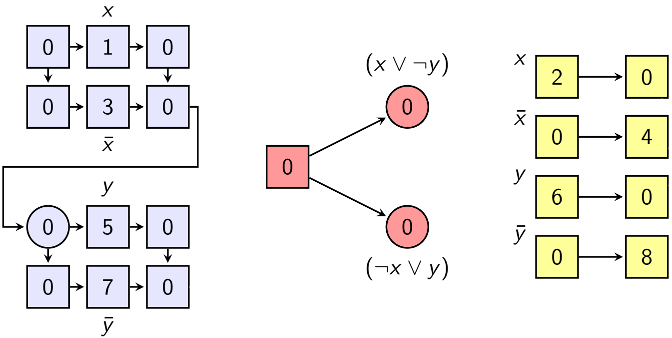

PSPACE-completeness

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The given decision problem is PSPACE*-hard.*

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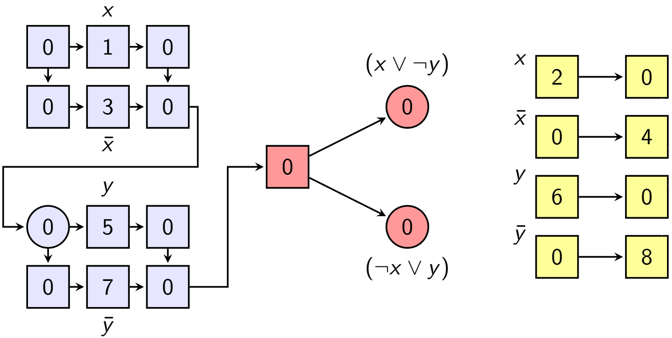

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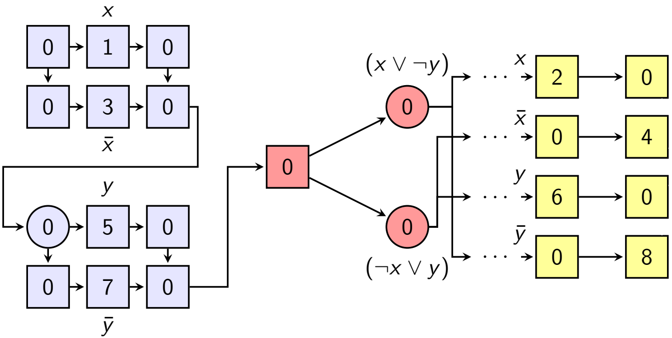

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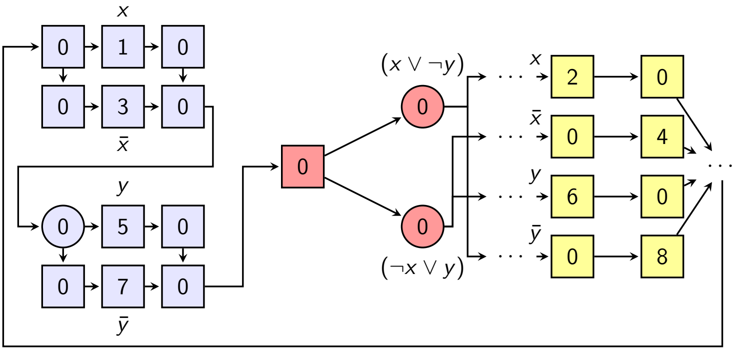

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PSPACE-hard

Exponential Memory

Sufficient

Necessary

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Memory Requirements (for Player 0)

Theorem

Optimal strategies for parity games require exponential memory.

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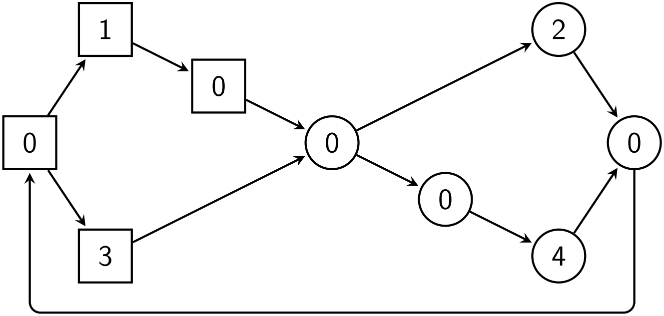

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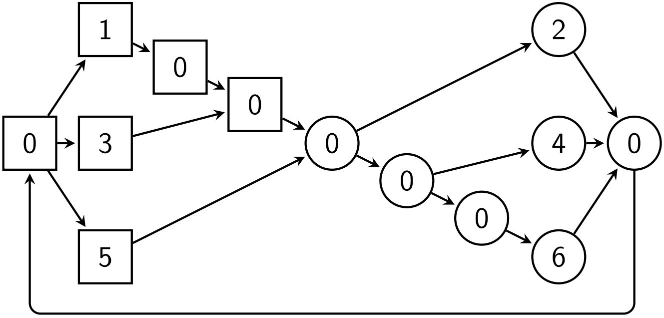

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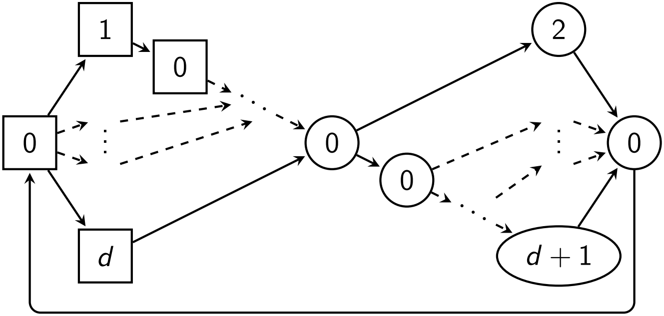

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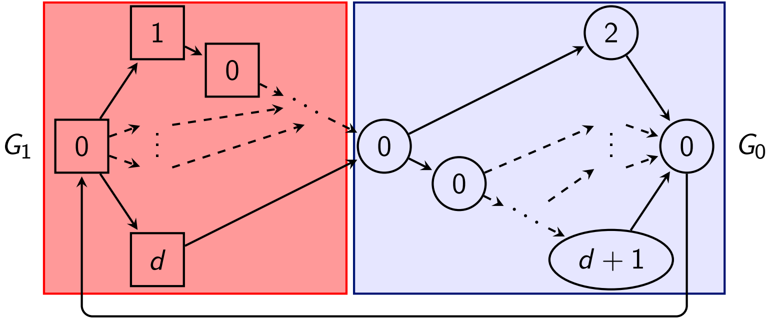

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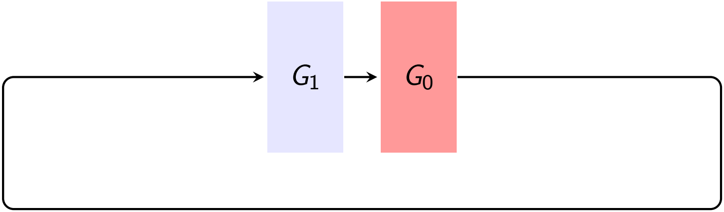

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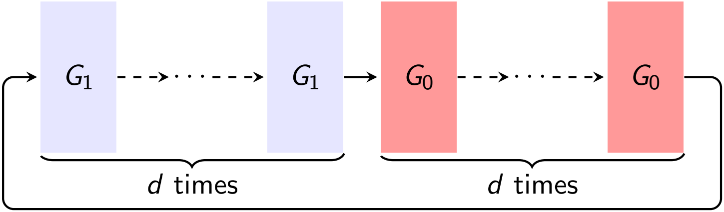

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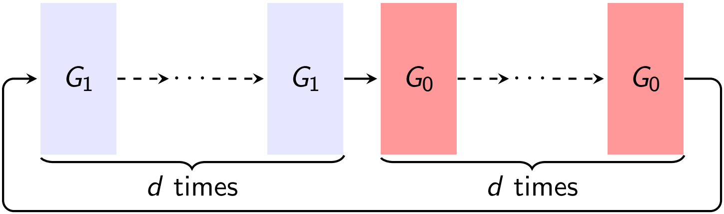

Player 0 needs to store d choices of d possible values each

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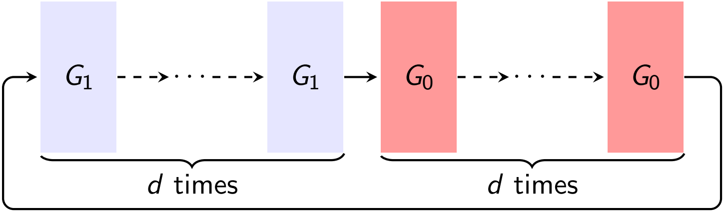
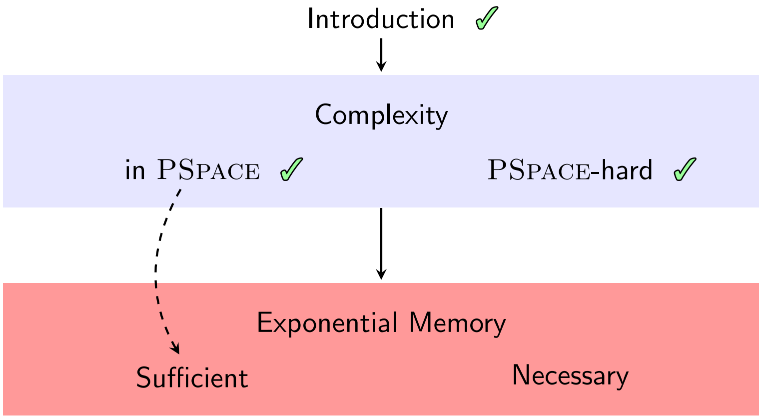


Player 0 needs to store d choices of d possible values each
$\Rightarrow$ Player 0 requires $\approx 2^d$ many memory states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Conclusion

	Parity
Complexity	$\mathrm{UP} \cap \mathrm{co\text{-}UP}$
Strategies	1

Conclusion

	Parity	Finitary Parity	
		Winning	
Complexity	$\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$	$\mathrm{P}\textsc{Time}$	
Strategies	1	1	

Conclusion

	Parity	Finitary Parity	
		Winning	Optimal
Complexity	$\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$	$\mathrm{P}\textsc{Time}$	$\mathrm{PS}\textsc{pace}$-comp.
Strategies	1	1	Exp.

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	Parity	Finitary Parity	
		Winning	Optimal
Complexity	$\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$	PTime	PSpace-comp.
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Take-away: Forcing Player 0 to answer quickly in parity games makes it harder

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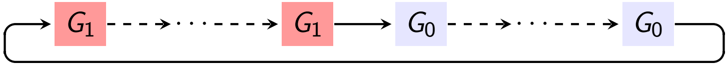
Tradeoffs

Parity Games with Costs

Tradeoffs



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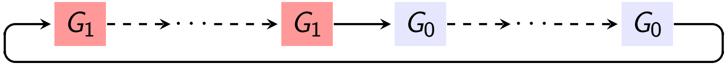
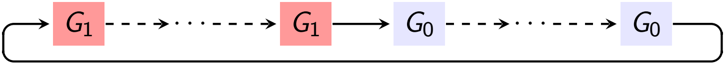


	Winning	Optimal
Size	1	2^d
Cost	$3d$	$2d$

Tradeoffs



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Size	1	d	2^d
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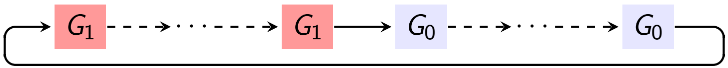
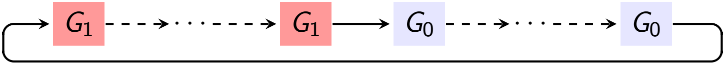
Tradeoffs



	Winning			Optimal
Size	1	d		2^d
Cost	$3d$	$3d-1$		$2d$

Tradeoffs



	Winning			Optimal
Size	1	d	2^{d-1}	2^d
Cost	$3d$	$3d-1$		$2d$

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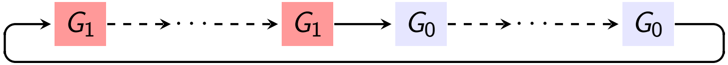
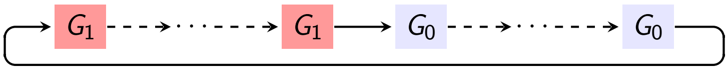


	Winning			Optimal
Size	1	d	2^{d-1}	2^d
Cost	$3d$	$3d-1$	$2d+1$	$2d$

Tradeoffs



	Winning				Optimal
Size	1	d	$\cdots$	2^{d-1}	2^d
Cost	$3d$	$3d-1$	$\cdots$	$2d+1$	$2d$

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Parity Games with Costs

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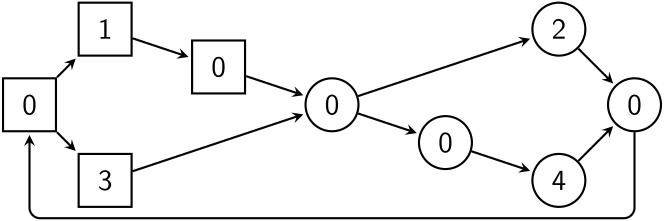

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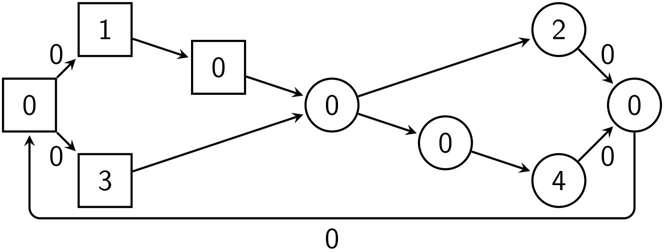

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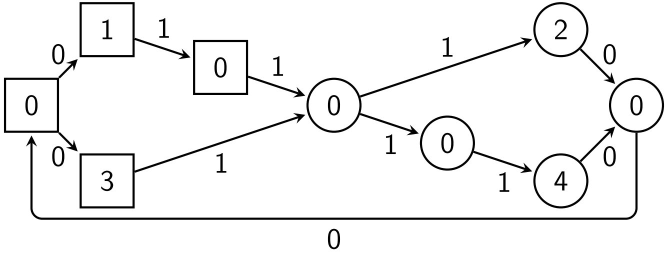

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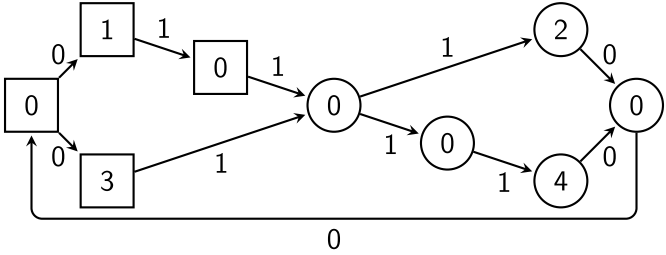

Finitary parity games are special case

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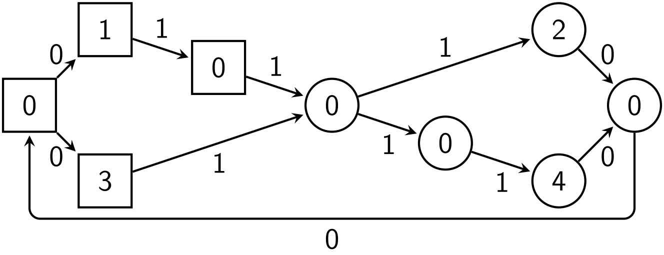

Finitary parity games are special case

$\Rightarrow$ PSPACE-hard $\Rightarrow$ Exp. memory necessary

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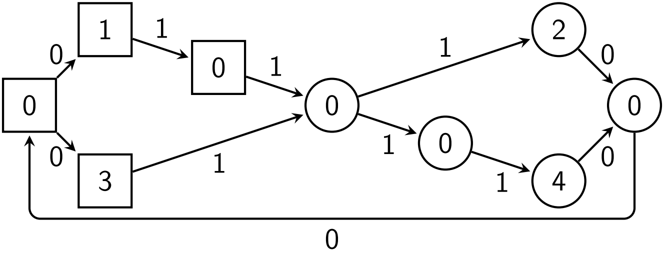

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Algorithm for solving finitary games works as well

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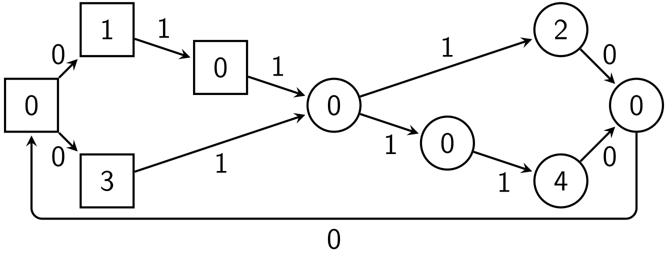

Finitary parity games are special case

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Algorithm for solving finitary games works as well

$\Rightarrow$ In $\mathrm{PS{\small PACE}}$ $\Rightarrow$ Exp. memory sufficient

Conclusion

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Conclusion

	Parity	Cost-Parity
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	Parity	Cost-Parity	
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Complexity	$\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$	$\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$	$\mathrm{PS}\textsc{pace}$-comp.
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